# AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES SCHOOLS ENRICHMENT CENTRE (AIMSSEC) <br> AIMING HIGH 

## MIRROR MIRROR

Investigate what happens if you reflect the blue flag in Mirror 1, then reflect that reflection in Mirror 2.

Do you get the same results for the red flag if you reflect it first in Mirror 2 then reflect that reflection in Mirror 1?

You might like to copy the diagram onto squared paper to help you with this investigation.

Describe the single transformation that takes the first flag to the third flag in this sequence of reflections?


Repeat this with flags in different positions and orientations, and with different pairs of parallel lines. What happens? Does it matter in which line you reflect first?

## HELP



Look at your own hands. What do you notice?
Look at yourself in a mirror. What do you notice?
Touch your right ear. What does your reflection do?


Think first about these reflections in one mirror line.
Draw the missing symbol to solve this puzzle
Draw some shapes, draw a mirror line and draw
 reflections of your shapes in the mirror line.

Use squared paper and draw the reflections as accurately as you can.

## NEXT

Summarise your findings about reflections in parallel mirrors in exactly 20 words (!) What would happen if the lines were not parallel?

If you have Geogebra you may be able to create dynamic geometry files that demonstrate this work.

## NOTES FOR TEACHERS



For the blue flags, the diagram shows B2 which is the reflection of the flag B1 in mirror line 1 and the flag B 3 which is the reflection of B 2 in the second mirror line. The single transformation that maps B1 to B3 is a translation from left to right perpendicular to the mirror lines by twice the distance between the mirror lines.

For the red flags, the diagram shows R 2 which is the reflection of the flag R1 in mirror line 2 and the flag R3 which is the reflection of R2 in mirror line 1.
The single transformation that maps R1 to R3 is a translation from right to left (the negative direction) perpendicular to the mirror lines by twice the distance between the mirror lines in.

This shows that the composition of two reflections in parallel mirror lines is a translation by twice the distance between the mirror lines but the direction of the translation depends on the order of the reflections.

## Why do this activity

This activity could be used as an extension task once learners have learnt to draw reflections accurately. It can provide a valuable mathematical challenge for learners who are ready to move on while others in the class need more practice with their basic drawing skills.
Alternatively, this problem and the two related problems on the NRICH website: ...on the Wall , and Who Is the Fairest of Them All? could form a unit of work on combined transformations. All three problems ask learners to consider the effect of combining two transformations, and then challenge them to describe the single transformations that produce the same results.

## Learning objectives

In doing this activity students will have an opportunity to:

- develop a better understanding of transformations;
- practise drawing accurate geometric constructions;
- develop visualization skills.


## Generic competences

In doing this activity students will have an opportunity to:

- Develop visualisation skills;
- Develop understanding of reflection in mirrors, which is important for example in looking at wing mirrors and rear view mirrors in a car.


## DIAGNOSTIC ASSESSMENT This should take about 5-10 minutes.

Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is $A, 2$ fingers for $B, 3$ fingers for $C$ and 4 for $D^{\prime \prime}$.

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.

3. Then do the same for answers $B, C$ and $D$. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up $1,2,3$ or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. As the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is: D
Possible misconceptions:
Tom has put the supposed reflection the same distance away from the mirror on the opposite side but not reflected it. Kate has reflected the object but failed to put it the same distance from the mirror line. https://diagnosticquestions.com

## Suggestions for teaching

As an extension task, simply provide the problem as given on page 1 to a pair of learners to work on together. When they have established the combined transformation for one specific example, a teacher intervention may be appropriate, to move the focus to the general case - asking the key questions below.

With a full class, encourage different learners to start with slightly different spacings of parallel lines and flag positions. The teacher intervention above could become a full class discussion. Students could regroup according to similarities/differences in the final combined transformations in order to acquire more information without needing to do a lot more drawing.

## Key questions

- What if the flag was in a different place?
- What if you reflected in the other line first?
- What if the lines were both at 45 degrees to the horizontal?
- What if the lines were both at 60 degrees to the horizontal? (use isometric paper)
- What, precisely, does the final position of the flag depend on?
- Can you prove it?


## Follow up

...on the wall https://nrich.maths.org/5459
Who is the fairest of them all? https://nrich.maths.org/5461
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\begin{array}{|ll|}\hline \text { MIATHS } & \begin{array}{l}\text { Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and } \\
\text { curriculum links: http://aiminghigh.aimssec.ac.za }\end{array}
$$ <br>

Subscribe to the MATHS TOYS YouTube Channel\end{array}\right\}\)| https://www.youtube.com/c/MathsToys/videos |
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| Download the whole AIMSSEC collection of resources to use offline with <br> the AIMSSEC App see $\underline{h t t p s: / / a i m s s e c . a p p ~ o r ~ f i n d ~ i t ~ o n ~ G o o g l e ~ P l a y . ~}$ |


| Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa. <br> New material will be added for Secondary 6. <br> For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339 <br> Mathematics taught in Year 13 (UK) \& Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12 |  |  |  |  |
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|  | Lower Primary Approx. Age 5 to 8 | Upper Primary Age 8 to 11 | Lower Secondary Age 11 to 15 | Upper Secondary Age 15+ |
| South Africa | Grades R and 1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| East Africa | Nursery and Primary 1 to 3 | Primary 4 to 6 | Secondary 1 to 3 | Secondary 4 to 6 |
| USA | Kindergarten and G1 to 3 | Grades 4 to 6 | Grades 7 to 9 | Grades 10 to 12 |
| UK | Reception and Years 1 to 3 | Years 4 to 6 | Years 7 to 9 | Years 10 to 13 |

