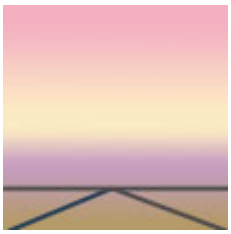
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

Title: INTERSECTIONS (Grades 9 and 10)

(1) Plot the graphs of the straight lines y = x,
y = 1.1x - 0.2 and y = 0.9x – 0.1
What do the graphs tell you about the solutions to the two pairs of simultaneous equations
y = x
y = 1.1x - 0.2
and
y = x
y = 0.9x – 0.1



(2) Solve the two pairs of simultaneous equations:

$$x + 0.99999y = 2.99999$$
$$0.99999x + y = 2.99998$$
and
$$x + 1.00001y = 2.99999$$
$$0.99999x + y = 2.99998.$$

Explain why the solutions are so different and yet the pairs of equations are nearly identical. Considering the geometrical properties of the lines helps to explain why this happens.

Solution

(1) See below for the graphs. You might prefer to plot all 3 graphs on the same axes.
The first pair of lines intersect at (2, 2) giving the solution x = 2, y = 2.
The second pair of equations intersect at (-1, -1) giving the solution x = -1, y = -1.
Although the equations are not very different the solutions are quite far apart because of the slightly different gradients.
(2) The solution to the first pair of equations is x = 2, y=1
The solution to the second pair of equations is x = - 199998, y = + 199999
For the first two lines one gradient is a little less than minus one and the other gradient a little more than minus one.
For the second two lines both gradients are a little more than minus one.
All four lines cut the *y*-axis very near to (0 , 3).
Because the pairs of lines in each case are nearly parallel the slight and change in the gradient of each line away from minus one causes the intersection to occur in very different places.

Notes for teachers

Why do this activity?

On the one hand this activity is a simple exercise in solving pairs of linear simultaneous equations but on the other it provides perhaps unexpected results that call for investigating the connection between the algebra and geometry, and considering the equations of the lines and gradients. It also requires some care in solving the equations because of the big numbers involved. Tasks on simultaneous equations can become familiar, routine experiences for students. This type of problem causes a "stop and think" moment.

Possible approach

Set this as homework or as a lesson starter and have a class discussion about the results.

Key questions

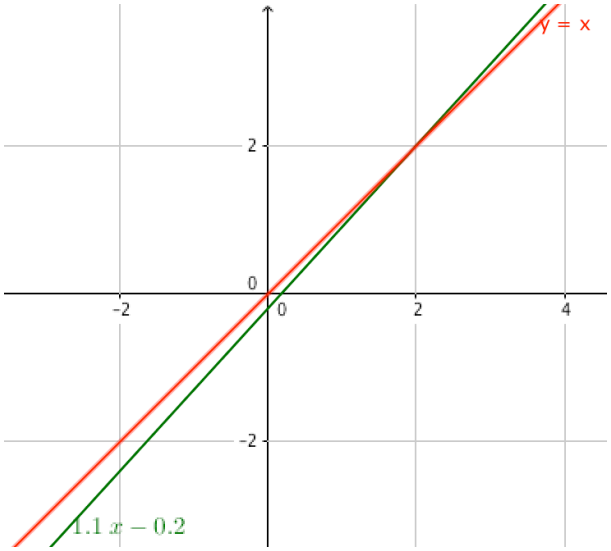
What are the gradients of the lines? (demonstrate with two pencils or sticks)
Are the pairs of lines nearly parallel or do they have very different gradients.
Where do the lines cut the y axis?
What happens if lines are nearly parallel and you change the gradients?
Why are the solutions of the two pairs of simultaneous linear equations so different when the equations are so nearly the same?

Possible Extension

Learners could be asked to make up their own similar example like the one given in the Possible Support section below.

Possible Support

Learners who have difficulty could be given these two graphs and pairs of simultaneous equations and asked:
What do you notice about these two graphs?
What do the graphs tell you about the solutions of the two pairs of simultaneous equations:



y = x
y = 1.1x – 0.2 and

y = x
y = 0.9x – 0.1 ?