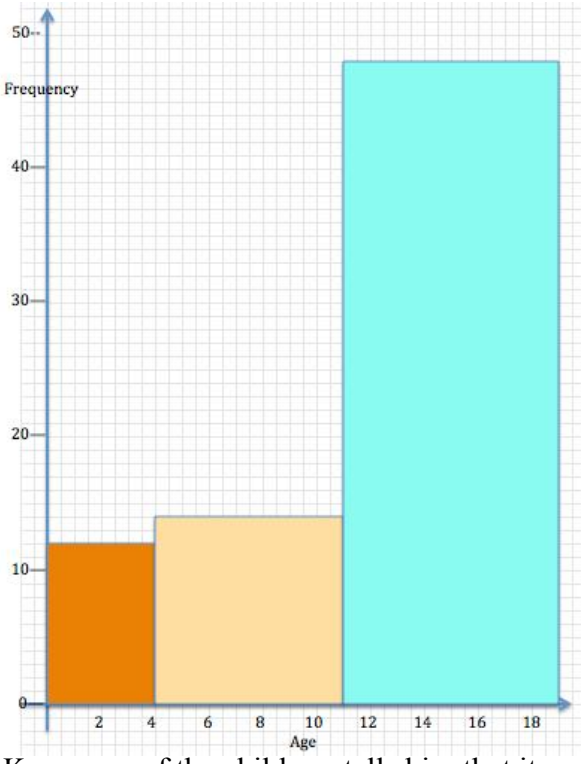
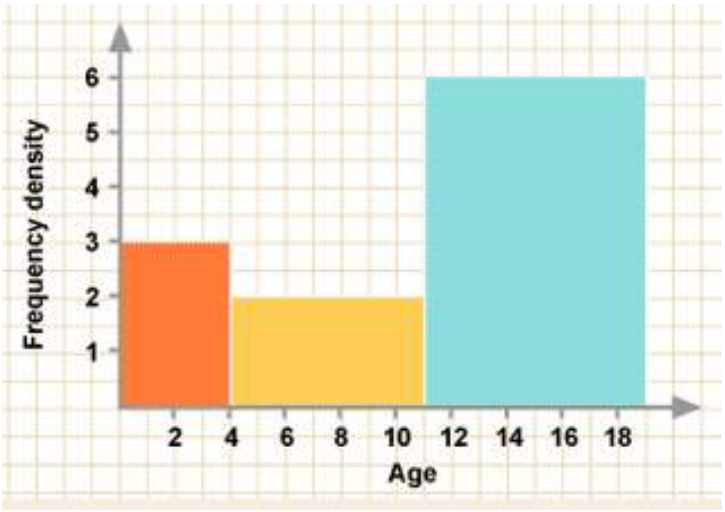
BEST REPRESENTATION





The ages of children arriving at a zoo in 1 hour are noted by the ticket salesman. There are 12 children between 0 and 3 years old, 14 children between 4 and 10 years old and 48 between 11 and 18 years old.

He draws a bar graph or frequency diagram like the one on the left to represent this data.

Kaya, one of the children, tells him that it would be better to represent the data in a histogram and she draws the diagram on the right.

Compare the two diagrams. What is the same and what is different about these two diagrams?

Explain how the frequencies are represented in each of the diagrams. Which is the best representation?

Do you agree with Kaya or not? Give reasons.

Help

You will probably find it helpful to work with a partner so that you can help each other to figure out what the question means and how to give your answers. Keep re-reading it trying to make sense of it and answer the questions. Have a good try, and don't be give up without a struggle because struggling with a problem is a good way to learn. But if you and your partner are both completely stuck, tell your teacher who may arrange for you to work with another learner or learners who will share their ideas with you.

Extension

The diagram on the right is called a histogram.

If you finish before other people in the class you and your partner could plan how to collect the data so that the class can draw a histogram for the ages of the brothers and sisters of children in the class. First work out the best way to collect the data and make a chart on which you will keep a record. Ask the teacher if you can collect this data from the other members of the class. If time you might plan how to draw and label the axes.

NOTES FOR TEACHERS

SOLUTION

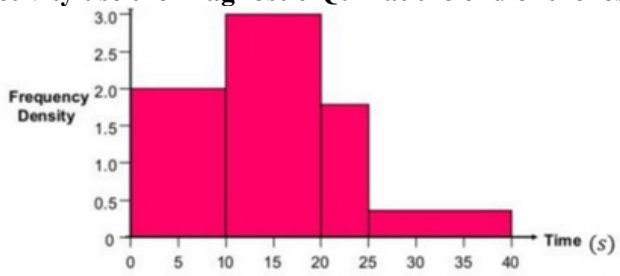
The boundaries for the groups are 0, 4, 11 and 19 because for example a child is considered to be 3 years old until his 4th birthday. The width of the class intervals are 4 years, 7 years and 8 years.

The heights in the frequency bar graph correspond to the frequencies of 12, 14 and 48. This would be suitable if the class widths were the same. With different class widths the visual effect of these bars is to magnify and exaggerate the differences in the frequencies. For example, 14 is only slightly more than 12 but the area of the corresponding bar is more than double.

Where the class widths are unequal the data should be represented in a histogram where the AREA of the bars represents the frequency. In this example 1 square unit represents a frequency of 1.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. **For this activity use the Diagnostic Quiz at the end of the lesson.**



This histogram shows the time taken by students to complete a task.

How many students took between 10 and 20 seconds?

A	B	C	D
30	3	3.333 ...	60

The correct answer is **A** because the area of the bar represents the frequency.

Misconceptions:

B. Students giving this answer have mistaken frequency density for frequency.

C. Students giving this answer have divided the time by the frequency density.

D. The student has multiplied by the upper boundary (20), rather than the class width.

https://diagnosticquestions.com

Why do this activity?

This problem provides an example of grouped data with unequal class intervals that learners can discuss. Even before they are introduced to the formal definition of a histogram they should be able to interpret these graphs and express opinions about the merits of the two representations. At the end of the discussion they must learn the definition of a histogram and they must realize that there are worldwide agreements about mathematical definitions so that everyone can have a common understanding.

Learning objectives

In doing this activity students will have an opportunity to make sense of the concept of representation of frequencies by areas in a histogram.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically**, reason logically and give explanations;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- evaluate, organise, analyse, and **interpret data**;
- **communicate** in writing, speaking and listening in pairs and as a whole class
    - exchange ideas, criticise, and present information and ideas to others
    - analyze, reason and record ideas effectively.

Suggestions for teaching

You could either use this activity as a way of introducing histograms or to review histograms. Tell the class that there will be a class discussion and they should be prepared to present their work on this activity to the class. To give learners practice in reading and interpreting questions for themselves, give them this activity without yourself giving any explanation or introduction. Ask the learners to work in pairs so that they can discuss the diagrams, what they think about them and whether they agree with Kaya or not.

If you see that any learners not making any progress then encourage them to keep re-reading all the information given trying to make sense of it. Ask key questions to focus their attention on the way forward. Encourage them to try again to answer the questions, and not to give up without a struggle because struggling with a problem is a good way to learn. But when you think help is needed you might arrange for the pair to work with another learner or learners.

Observe the learners at work so that you have a good idea of who to invite to give presentations to the class. Draw both diagrams on the board or use a projector so that everyone in the class can see them. Ask several learners to present their work. Ask the class who prefers the frequency bar chart and why. Ask who prefers the histogram and ask why. Guide the learners to see that areas accurately show the relative sizes of the 3 groups, that is 12, 14 and 48 children, whereas, because of the different widths of the bars, the other diagram shows these numbers out of proportion to each other.

Finally use the Diagnostic Quiz as formative assessment to see whether the learners have understood the concept. To reinforce the understanding of the concept ask other questions based on the histogram in the Diagnostic Quiz. **Explain that histograms are the universal standard way to represent this type of data.**

Key questions

- Can you read the ages at the edges of the bars in the graphs (the class boundaries)?
- Why do you think that 4 is the class boundary between the 0 to 3 years group and the 4 to 10 years group?
- How do you find the frequencies from the frequency bar graph on the left?
- How do you find the frequencies from the histogram on the right?
- Which diagram best represents the numbers 12 and 14 of the two groups of younger children without making one look hugely different from the other?

Follow up

South African Demographics https://aiminghigh.aimssec.ac.za/years-9-12-sa-demographics/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**