AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

This INCLUSION AND HOME LEARNING GUIDE

suggests related learning activities for all ages from 4 to 18

on the theme of SCALE

Choose what seems suitable for the age or attainment level of your learners

The original ENLARGEMENT was designed for Years 9 to 10

ENLARGEMENT

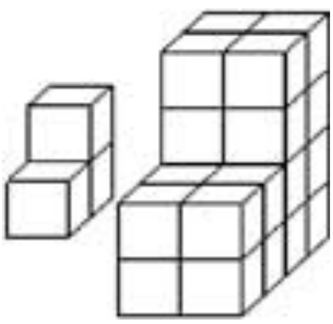
Make enlargements of the coloured shapes by fitting together four identical smaller shapes. Cut out the pieces to make the 3 puzzles.

How much longer are the edges (the linear scale factor of the enlargement)?

How much bigger is the area (the area scale factor of the enlargement)?

Call the small shapes Y_0, R_0, and B_0 and call the enlargements Y_1, R_1, and B_1.

Can you enlarge the shapes again by putting four of the Y_1, R_1, and B_1 pieces together to make Y_2, R_2, and B_2? What are the scale factors of the enlargements from Y_0, R_0, and B_0 to Y_2, R_2, and B_2?



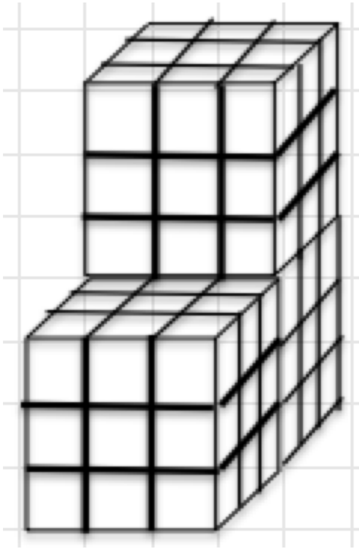
Look at the illustration of the 3-dimensional (3D) solids. You could call these 3D versions of the yellow reptile above. The coloured shapes are called reptiles because the enlargements can be repeated again and again to tile a flat surface.

One of the 3D models shown in the diagram is an enlargement of the other. Make these models. if you have some cubes.

What is the linear scale factor of the enlargement?

How much bigger is the surface area (the area scale factor)?

How much bigger is the volume (the volume scale factor)?



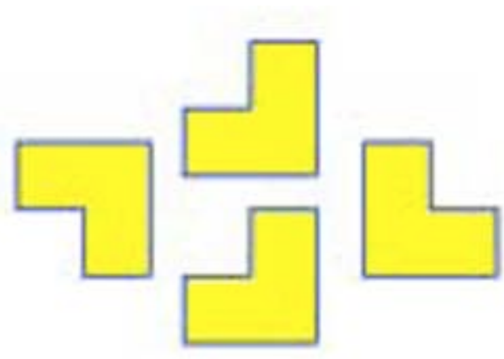
This diagram shows an enlargement of the 3D model with linear scale factor 3.

What are the area and volume scale factors?

HELP

Either draw 4 pieces of each shape on squared paper or print the shapes on page 3. Cut out the pieces to make the 3 puzzles.

NEXT



These 'trisquares' are made up of three squares and each has an area of 3 square units.

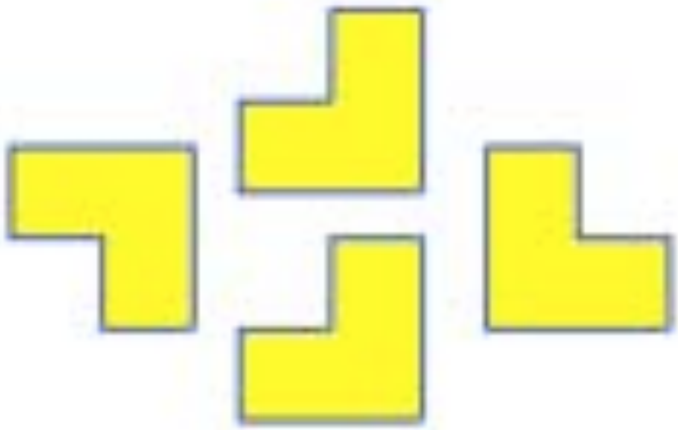
Can you fit them together to make an enlargement of the shape? What is its area?

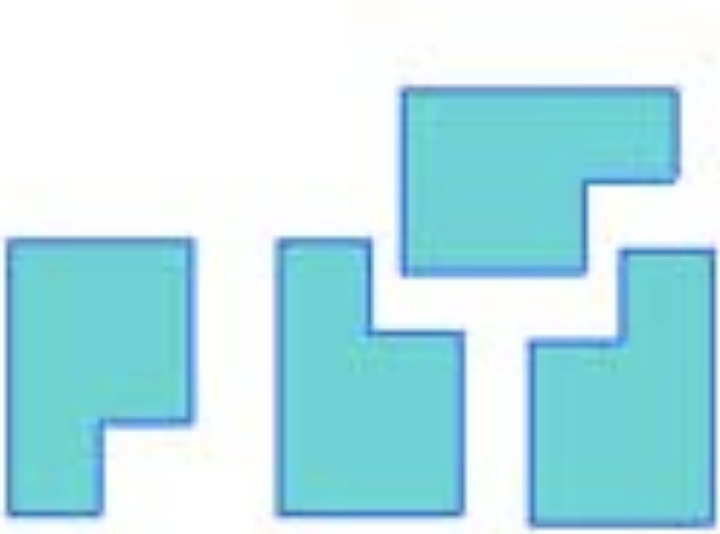
Can you fit trisquares together to make enlargements of scale factors 3, 4 and 5? What are their areas?

Is it possible to make enlargements of all sizes by fitting trisquares together?

Squared paper would be useful for working on this challenge.

See the activity TRISQUARES https://aiminghigh.aimssec.ac.za/years-9-12-trisquares





INCLUSION AND HOME LEARNING GUIDE
THEME: SCALE

Early Years





Talk about these pictures.
If possible, print them and cut them out. Even better make multiple copies and make cards with them.

How many pictures of Pooh Bear?
What is the same about them?
What is different about them?
Can you smile like Pooh Bear? Show me! Let's all smile.
What is Pooh Bear wearing? Who is he pretending to be?
What is Pooh Bear doing?
What colour is Pooh Bear? What colour is the suit he is wearing?
Which is the biggest?
Which is the smallest?
Arrange the pictures in order of size from smallest to biggest. What do you notice?
Arrange the pictures in order of size from biggest to smallest. What do you notice?
How many smaller pictures of Pooh Bear do you need to cover one of the bigger pictures?
There is a lot of scope in the last question for investigation and discovery.

Lower Primary Years





Talk about these pictures.
If possible, print them and cut them out. Even better make multiple copies and make cards with them.

We call these pictures **ENLARGEMENTS** of each other.

What is the same and what is different about these pictures of Pooh Bear?
What is Pooh Bear wearing? What is Pooh Bear doing? Who is he pretending to be?
What colour is Pooh Bear? What colour is the suit he is wearing?

Which picture is the biggest? And which is the smallest?
Pooh belongs to a little boy called Christopher Robin. How big do you think Pooh Bear actually is? How many times bigger is the real Pooh than the biggest picture here?

Arrange the pictures in order of size from smallest to biggest. What do you notice?
Arrange the pictures in order of size from biggest to smallest. What do you notice?

How many smaller pictures of Pooh Bear do you need to cover one of the bigger pictures?

There is a lot of scope in the last question for investigation and discovery.

Upper Primary

If they have not already done the Lower Primary activities, Upper Primary learners could do them as they are very suitable for these older learners. They should do an investigation to find out the scale factors for lengths by measuring the edges of the pictures. To discover the area scale factors they should place copies of one picture on top of another to see the relative areas covered. Notice that 4 of the pictures are enlarged by scale factors of 1, 2, 4 & 8. The other picture is scaled up by a factor of 1.5

LITTLE MAN

Little Man is much smaller than you and me. Here is a picture of him standing next to an ordinary mug.

Can you estimate how tall he is? How tall do you think Little Man's mug might be?

My mug is 10 centimetres tall and it holds 300 millilitres but mugs come in different sizes. How big is your mug?

Can you estimate how many "Little Man mugs" of tea might fill one of our mugs? For Little Man's mug in your investigation you might use a bottle top, or you might ask Mum or Granny if you could borrow their sewing thimble.

How many 5 millilitre spoonfuls do you think it would it take to fill Little Man's mug? It is important to know about millilitre measures which are used in prescribing medicines

See https://aiminghigh.aimssec.ac.za/years-4-6-little-man/

Lower Secondary – do part or all of the tasks on pages 1, 2 & 3

Make enlargements of the coloured shapes by fitting together four identical smaller shapes. Cut out the pieces to make the 3 puzzles.

How much longer are the edges (the linear scale factor of the enlargement)?

How much bigger is the area (the area scale factor of the enlargement)?

Call the small shapes Y_0, R_0, and B_0 and call the enlargements Y_1, R_1, and B_1.

Can you enlarge the shapes again by putting four of the Y_1, R_1, and B_1 pieces together to make Y_2, R_2, and B_2?

What are the scale factors of the enlargements from Y_0, R_0, and B_0 to Y_2, R_2, and B_2?

Question A: Can this process of **scaling up** go on for ever with the shapes made up of smaller and smaller copies inside themselves of the same shape?

Question B: Can a similar process of **scaling down** go on for ever with the shapes made up of smaller and smaller copies inside themselves of the same shape?

If the answers to **Questions A & B** are both yes, then we are seeing the stages of building up a **FRACTAL.** It would only be a true FRACTAL if we could continue this process for ever.

Years 9 and 10 – the tasks on pages 1, 2 & 3

You may like to give the pieces of the puzzle (see page 3) to pairs of learners so that they can cut them out and fit them together. Alternatively, the learners could be given squared paper on which to draw the enlargements and draw the copies of the smaller pieces fitting together inside.

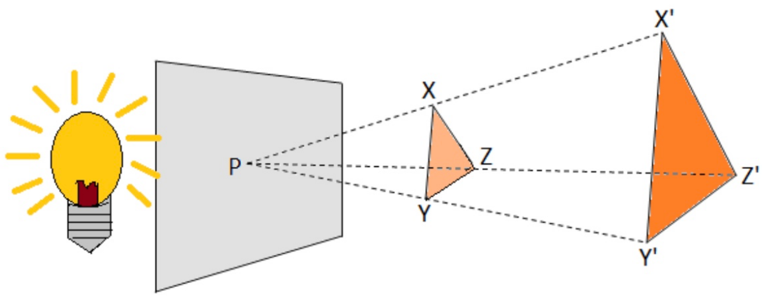
Alternatively, you may like to give out **one** large shape of each type for the learners to draw around in their notebooks to produce the solution outline and ask them to work backwards and to try to find the way that the 4 smaller pieces go together to make the larger copy. This type of question is called an 'inverse problem'.

When sharing ideas as a class you could use one very large shape of each type that can be drawn around on the board to show the solutions.

For the 3D models you should use cubes if possible.

These two diagrams are useful for explaining enlargement in 2D and in 3D and in relating the school maths to everyday experiences of enlargements when images appear on screen and when using a magnifying glass.

But the challenge here is "what sort of magnifying glasses are the little people using?"



Key questions

- How many times longer are the edges.
- What is the surface area?
- How many cubes make up the model? How does that give you the volume?
- How many times bigger is this surface area compared to that one?
- How many times bigger is this volume compared to that one?
- What do you notice about the numbers 2, 4, 8 and the numbers 3, 9, 27?

Upper Secondary

Do the learning activities on pages 1 to 3.
Call the small shapes Y_0, R_0, and B_0 (labelling them by colour) and call the enlargements Y_1, R_1, and B_1.

Can you enlarge the shapes again by putting four of the Y_1, R_1, and B_1 pieces together to make Y_2, R_2, and B_2?

What are the scale factors of the enlargements from Y_0, R_0, and B_0 to Y_2, R_2, and B_2?

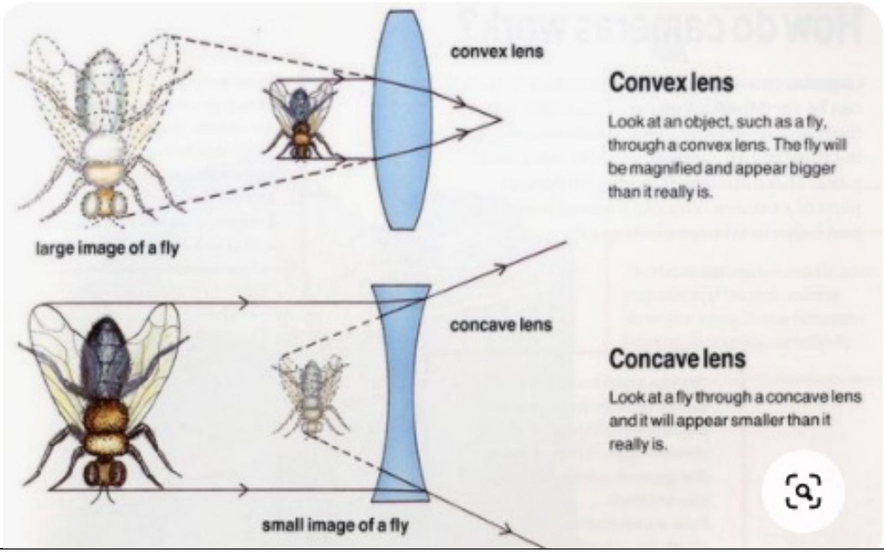
Question A: Can this process of **scaling up** go on for ever with the shapes made up of smaller and smaller copies of the same shape inside themselves?

Question B: Can a similar process of **scaling down** go on for ever with the shapes made up of smaller and smaller copies of the same shape inside themselves?

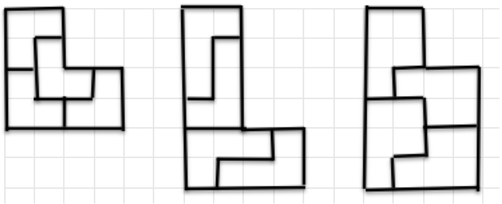
If the answers to **Questions A & B** are both yes, then we are seeing the stages of building up a **FRACTAL.** It would only be a true FRACTAL if we could continue this process for ever.

Study these pictures show how convex and concave lenses bend the light that enters the eye so that one lens magnifies the subject and the other lens reduces it in size. Which lens are the little people using in the picture below.

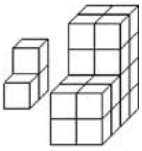
These lenses are used in glasses and contact lenses to correct our vision.



SOLUTION



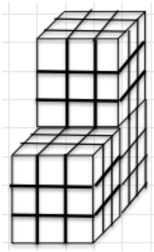
The edge lengths are doubled in each of these enlargements.
The linear scale factor is 2 and
The area scale factor is 4.



For the cubes illustrated, the smaller model has a surface area of 14 square units and it is made with 3 cubes.

The larger model has a surface area of 56 = 14 x 4 square units and it is made with 24 = 3 x 8 cubes .

The linear scale factor is 2, the area scale factor is 4 and the volume scale factor is 8.

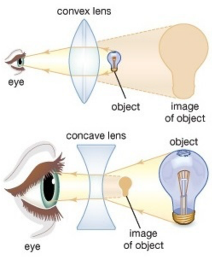


This diagram shows an enlargement with linear scale factor 3.

The surface area is 126 = 14x9 square units so the area scale factor is 9.

The volume is 81 = 3 x 27 cube units so the volume scale factor 27.

In this picture the little people are not using the usual sort of magnifying glass they are using concave lenses that make the images smaller.



Why do this activity?

This activity starts with simple puzzles and then it gives learners practice in working out surface areas and volumes and connecting the concept of similarity with enlargement and scale factors.

Learning objectives

In doing this activity students will have an opportunity to:

- learn about scale and scale factors
- deepen their understanding of enlargement of 2D and 3D shapes
- develop visualization skills

Generic competences

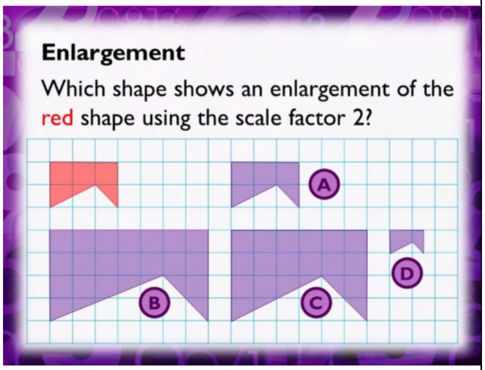
In doing this activity students will have an opportunity to:

- gain a better understanding of similarity and enlargement in their surroundings and in applications to design and representation in practical contexts;
- develop visualization skills that can be applied to reading maps and other practical applications.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Use it either at the start or at the end of a lesson. Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.

The correct answer is: **C**

Common Misconceptions

A. Learners have chosen the congruent shape.

B. This diagram shows an enlargement of scale factor $\frac{7}{3}$.

D. Again a poor understanding, This diagram shows an enlargement by scale factor $\frac{1}{2}$.

https://diagnosticquestions.com

https://diagnosticquestions.com

Follow up

Little Man https://aiminghigh.aimssec.ac.za/years-4-6-little-man/
Trisquares https://aiminghigh.aimssec.ac.za/years-9-12-trisquares
Shadows https://aiminghigh.aimssec.ac.za/years-9-10-enlargement/
Sierpinski Number and Shape Patterns
https://aiminghigh.aimssec.ac.za/grades-6-12-sierpinski-number-and-shape-patterns/
Seven Squares https://aiminghigh.aimssec.ac.za/years-7-8-seven-squares/

MATHS TOYS

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel** https://www.youtube.com/c/mathstoys
Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App see https://aimssec.app Find the App on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13