

### AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES

SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

### **AIMING HIGH**

# **POWERFUL THINKING 4**

Is this number  $3^{444} + 4^{333}$  divisible by 5?

2<sup>666</sup> + 6<sup>222</sup> 3<sup>444</sup> + 4<sup>333</sup> 1<sup>999</sup> + 9<sup>111</sup>

Investigate other big powers?

Make up some similar numerical expressions involving powers that have interesting properties.

What about 2<sup>666</sup> + 6<sup>222</sup>? 1<sup>999</sup> + 9<sup>111</sup>? 7<sup>444</sup> + 4<sup>777</sup>?

Create some patterns of your own involving the sum of two (or perhaps three) big powers, and describe their special properties.

# HELP

What pattern can you see in the last digits of the powers of 2? What pattern can you see in the last digits of the powers of 6? Now add  $2^2 + 6^2$ ,  $2^3 + 6^3$ ,  $2^4 + 6^4$ ,... but just focus on the last digits. Make a list of the last digits that you get. Try to spot a pattern and how the pattern keeps repeating itself. Where does 222 occur in that pattern?

For the second part of this question, what pattern can you see in the last digits of the powers of 3? Where does 444 occur in that pattern?

What pattern can you see in the last digits of the powers of 4? Where does 333 occur in that pattern?

Now add  $3^4 + 4^3$ ,  $3^8 + 4^6$ ,  $3^{12} + 4^9$ ,... but just focus on the last digits. Make a list of the last digits that you get. Try to spot a pattern and how the pattern keeps repeating itself.

# NEXT

Make up a similar expression involving sums or products of powers of whole numbers. You might include sums of powers of three numbers. Write down a list of the properties that you see.

Exchange with another learner list the properties of the expressions that your partner has created. Then discuss the properties and between you find the best way to describe and explain the properties.



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# Notes for teachers

Solution												
The last digits of powers	1	Last digits for n <sup>th</sup> powers										
of 2 repeat in 4-cycles, 2486 repeated (see	Base	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10	
highlighting).	1	1	1	1	1	1	1	1	1	1	1	
As $666 = (4x166) + 2$ so	2	<mark>2</mark>	<mark>4</mark>	<mark>8</mark>	6	<mark>2</mark>	<mark>4</mark>	<mark>8</mark>	6	<mark>2</mark>	<mark>4</mark>	
the 4-cycles repeat 166 times then $2^{665}$ ends in 2	3	3	9	7	1	3	9	7	1	3	9	
	4	4	6	4	6	4	6	4	6	4	6	
and $2^{666}$ ends in 4.	5	5	5	5	5	5	5	5	5	5	5	
Note that the last digits of powers of 6 are always 6 so the last digit of $2^{666} + 6^{222}$ is 0 because 4 + 6 = 10	6	6	6	6	6	6	6	6	6	6	6	
	7	7	9	3	1	7	9	3	1	7	9	
	8	8	4	2	6	8	4	2	6	8	4	
	9	9	1	9	1	9	1	9	1	9	1	
4 + 0 - 10.												

So  $2^{666} + 6^{222}$  is divisible by 10 (or we can say it is a multiple of 10).

Similarly, the last digits of powers of 3 repeat in 4-cycles of 3971 repeated over and over again. Every  $4^{th}$  last digit is 1 so the last digit of  $3^{444}$  is 1 as 444 is divisible by 4.

The last digits of powers of 4 repeat in 2-cycles of 46 repeated, and all the last digits of odd powers are 4 so the last digit of  $4^{333}$  is 4 as 333 is an odd number.

So the last digit of  $3^{444} + 4^{333}$  is 1 + 4 = 5 and  $3^{444} + 4^{333}$  is divisible by 5 (or we can say it is a multiple of 5).

Again there is a cyclic pattern in last digits of powers of 9 which repeat in 2-cycles, 91 repeated over and over again, and so the last digit 9<sup>111</sup> is 9 as 111 is an odd number

Note that the last digits of powers of 1 are always 1 so the last digit of  $1^{999} + 9^{111}$  is 0 because 9 + 1 = 10. So  $1^{999} + 9^{111}$  is divisible by 10 (or we can say it is a multiple of 10).

The last digits of powers of 7 repeat in 4-cycles 7931 so the last digit of 7<sup>444</sup> is 1 as 444 is divisible by 4 and every 4<sup>th</sup> term in this cycle is 1.

The last digits of powers of 4 repeat in 2-cycles, with 46 repeated, and all the last digits of odd powers are 4 so the last digit of  $4^{777}$  is 4 as 777 is an odd number.

So the last digit of  $7^{444} + 4^{777}$  is 1 + 4 = 5 and  $7^{444} + 4^{777}$  is divisible by 5 (or we can say it is a multiple of 5).

Also  $6^{888} + 8^{666}$  is divisible by 10 and  $3^{777} + 7^{333}$  is divisible by 10.

If you investigate powers you will find other patterns.



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**Diagnostic Assessment** This should take about 5–10 minutes.

- 1. Write the question on the board, say to the class: "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
- 2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- 3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- 4. Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
- 5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The powers of 3 are  $3^{0}=1, 3^{1}=3, 3^{2}=9, 3^{3}=27, 3^{4}=81, 3^{5}=243, 3^{6}=729, 3^{7}=2187, 3^{8}=6561, 3^{9}=19683, ...$ Can you see a pattern in the last digits of these powers? Use this pattern to decide which of the following choices is the LAST DIGIT of  $3^{20}$ . A. 1 B. 3 C. 7 D. 9 The correct answer

The correct answer is A because the last digits repeat in a 4-cycle

1-3-9-7-1-3-9-7-1, .... and when the power is a multiple of 4 the last digit is 1.

https://diagnosticquestions.com

# Why do this activity

This learning activity, working with big powers, requires learners to think mathematically about more general number patterns and it can be used to review, give further practice, and reinforce understanding of the operational rules and concepts relating to exponents. The reasoning involves clock arithmetic (modular arithmetic). Learners are familiar with clock arithmetic through clocks, days of the week, months of the year etc. so this can also be discussed in class. It is the fourth in a set of similar activities and the earlier ones can be used for learners needing more support while the open ended nature of this activity provides a challenge for more able learners to create their own number patterns involving exponents.

Learners may find number patterns of this type pleasing and you may like to make their own poster for the classroom wall displaying a variety of patterns and noting the properties. Click here for a poster for this activity <u>http://nrich.maths.org/6176</u>

# Learning objectives

In doing this activity students will have an opportunity to:

- investigate large powers and the cyclic behavior in the last digits of powers;
- make and prove conjectures about powers (exponents).

### **Generic competences**

In doing this activity students will have an opportunity to:

- think flexibly, be creative and innovative and apply knowledge and skills;
- persevere and work systematically to investigate all possible cases;
- **communicate** arguments in a clear, concise and accurate way.



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## Suggestions for teaching

Start with the diagnostic quiz as a warm up, to help to assess the learners' prior knowledge of powers (exponents) and to prepare them to understand what the question is asking.

The best approach is to give them this question in written form (on the board or on a worksheet) and tell them to work individually to solve it. In tests they need to be able to read and answer the questions without help and they need lots of practice in class of doing the same.

Give learners the time to think about how to use what they already know to tackle this problem, and encourage them to think for themselves rather than being dependent on you, as the teacher, to tell them what to do.

Observe how the learners are progressing and ask questions to guide individuals if they need some support and encouragement. You may choose to give them the table above and suggest that they fill it in to help them in their work. Learners may naturally record their results in a table if they have often done so before.

Then ask the learners to work in pairs to compare methods and answers and then to continue working on the problem. The time that you allow for individual work and for working in pairs will depend on your class. If you have a very large class, and limited space for you to get around the room to help individuals, then working in pairs enables learners to help each other and you could organize seating arrangements to optimize this. Checking each other's work is efficient as learners can help each other to correct mistakes, but teachers should collect in, check and write in the learner's books on a regular basis (even if not frequently).

In the last part of the lesson ask learners to come to the board and explain what they have found out. Check that the reasoning is correct and provide a summary of what has been learned. Creating their own patterns could be a homework or follow-up task in the next lesson.

### **Key Questions**

- What do you notice?
- What do you notice about the last digits?
- Can you explain how you know that?
- Can you describe a pattern that you see?
- Can you see a repeating pattern? How many digits are repeated?
- What is the length of the cycle of days of the week? (or months of the year?)
- How many cycles of length 4 would you complete before you get to the 666<sup>th</sup> power (or the 444<sup>th</sup>)?

### Follow up

See Powerful Thinking 1 <u>https://aiminghigh.aimssec.ac.za/years-7-10-powerful-thinking-1/</u> and Powerful Thinking 2 <u>https://aiminghigh.aimssec.ac.za/years-8-10-powerful-thinking-2/</u> and Powerful Thinking 2 <u>https://aiminghigh.aimssec.ac.za/years-9-10-powerful-thinking-3/</u>

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.							
	Lower Primary or	Upper Primary	Lower Secondary	Upper Secondary			
	Foundation Phase						
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12			
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12			
UK	Reception and Years 1 to	Years 4 to 6	Years 7 to 9	Years 10 to 13			
East Africa	Nursery and Primary 1 to	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6			