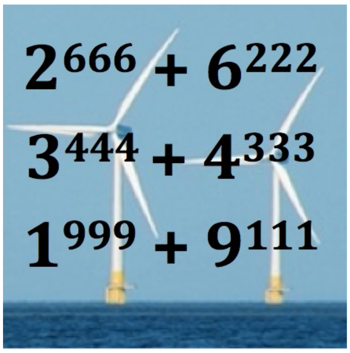
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

POWERFUL THINKING 4



Is this number $3^{444} + 4^{333}$ divisible by 5?

Investigate other big powers?

Make up some similar numerical expressions involving powers that have interesting properties.

What about
$2^{666} + 6^{222}$?
$1^{999} + 9^{111}$
$7^{444} + 4^{777}$?

Create some patterns of your own involving the sum of two (or perhaps three) big powers, and describe their special properties.

Solution

The last digits of powers of 3 repeat in 4-cycles of 3971 repeated. Every 4th last digit is 1 so the last digit of 3^{444} is 1 as 444 is divisible by 4.

The last digits of powers of 4 repeat in 2-cycles of 46 repeated, and all the last digits of odd powers are 4 so the last digit of 4^{333} is 4 as 333 is an odd number.

Last digits for n^{th} powers									
n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10
1	1	1	1	1	1	1	1	1	1
2	4	8	6	2	4	8	6	2	4
3	9	7	1	3	9	7	1	3	9
4	6	4	6	4	6	4	6	4	6
5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6
7	9	3	1	7	9	3	1	7	9
8	4	2	6	8	4	2	6	8	4
9	1	9	1	9	1	9	1	9	1

So the last digit of $3^{444} + 4^{333}$ is 1 + 4 = 5 and **$3^{444} + 4^{333}$ is divisible by 5** (or we can say it is a multiple of 5).

The last digits of powers of 2 repeat in 4-cycles, 2486 repeated, and so the last digit 2^{666} is 4 as

666=(4x166) + 2 so the 4-cycles repeat 166 times then 2^{665} ends in 2 and 2^{666} ends in 4.

Note that the last digits of powers of 6 are always 6 so the last digit of $2^{666} + 6^{222}$ is 0 because 4 + 6=10. So **$2^{666} + 6^{222}$ is divisible by 10** (or we can say it is a multiple of 10).

The last digits of powers of 9 repeat in 2-cycles, 91 repeated, and so the last digit 9^{111} is 9 as 111 is an odd number

Note that the last digits of powers of 1 are always 1 so the last digit of $1^{999} + 9^{111}$ is 0 because 9 + 1=10. So **$1^{999} + 9^{111}$ is divisible by 10** (or we can say it is a multiple of 10).

The last digits of powers of 7 repeat in 4-cycles 7931 so the last digit of 7^{444} is 1 as 444 is divisible by 4 and every 4th term in this cycle is 1.
The last digits of powers of 4 repeat in 2-cycles, with 46 repeated, and all the last digits of odd powers are 4 so the last digit of 4^{777} is 4 as 777 is an odd number.
So the last digit of $7^{444} + 4^{777}$ is 1 + 4 = 5 and **$7^{444} + 4^{777}$ is divisible by 5** (or we can say it is a multiple of 5).

Also $6^{888} + 8^{666}$ is divisible by 10 and $3^{777} + 7^{333}$ is divisible by 10. You might find other patterns.

Notes for teachers

Why do this activity

This learning activity, working with big powers, requires learners to think mathematically about more general number patterns and it can be used to review, give further practice, and reinforce understanding of the operational rules and concepts relating to exponents. The reasoning involves clock arithmetic (modular arithmetic) with which learners are familiar through clocks, days of the week, months of the year etc. so this can also be discussed in class. It is the fourth in a set of similar activities and the earlier ones can be used for learners needing more support while the open ended nature of this activity provides a challenge for more able learners to create their own number patterns involving exponents.

Learners may find number patterns of this type pleasing and you may like to make a poster for the classroom wall displaying a variety of patterns and noting the properties. Click here for a poster for this activity http://nrich.maths.org/6176

Possible approach

If your teaching style is to give learners the time to think about how to use what they already know to tackle a problem, and to encourage them to think for themselves rather than being dependent on you, as the teacher, to tell them what to do, then the best approach is to give them this question in written form (on the board or on a worksheet) and tell them to work individually to solve it. In tests they need to be able to read and answer the questions without help and they need lots of practice in class of doing the same.

Observe how the learners are progressing and ask questions to guide individuals if they need some support and encouragement. You may choose to give them the table above and suggest that they fill it in to help them in their work. Learners may naturally record their results in a table if they have often done so before.

Then ask the learners to work in pairs to compare methods and answers and then to continue working on the problem. The time that you allow for individual work and for working in pairs will depend on your class. If you have a very large class, and limited space room for you to get around the room to help individuals, then working in pairs enables learners to help each other and you could organize seating arrangements to optimize this. Checking each other's work is efficient as learners can help each other to correct mistakes, but teachers should collect in, check and write in the learner's books on a regular basis (even if not frequently).

In the last part of the lesson ask learners to come to the board and explain what they have found out. Check that the reasoning is correct and provide a summary of what has been learned. Creating their own patterns could be a homework task.

Key Questions

What do you notice?
What do you notice about the last digits?
Can you explain how you know that?
Can you describe a pattern that you see?
Can you see a repeating pattern? How many digits are repeated?
What is the length of the cycle of days of the week? (or months of the year?)
How many cycles of length 4 would you complete before you get to the 666^{th} power (or the 444^{th})?

Possible Extension

Learners can make up a similar expression involving sums or products of powers of whole numbers, including sums of powers of three numbers, and write down a list of the properties that they see. They can exchange their work with another learner and check that properties are correctly described.

Possible Support

See Powerful Thinking 1 https://aiminghigh.aimssec.ac.za/grade-7-powerful-thinking-1/
and Powerful Thinking 2 https://aiminghigh.aimssec.ac.za/grades-8-10-powerful-thinking-2/
and Powerful Thinking 2 https://aiminghigh.aimssec.ac.za/grades-8-10-powerful-thinking-3/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA and to Years 4 to 12 in the UK.

	Lower Primary or Foundation Phase	Upper Primary	Lower Secondary	Upper Secondary
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6