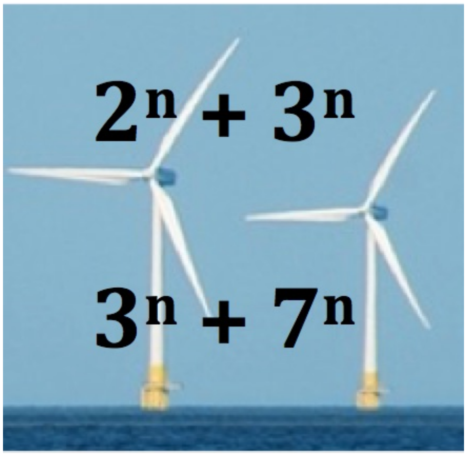
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

AIMING HIGH

POWERFUL THINKING 2



Will the number $2^n + 3^n$ be a power of 10 for any value of n?

For which values of n is this number divisible by 5?

What can you say about the values of n that make the second number $3^n + 7^n$ a power of 10?

Are there any other pairs of integers between 1 and 10 that have similar properties?

What other patterns with powers can you find that have special properties?

HELP

To record your findings you could fill in the table below. Just think about the last digit (the units digit) each time.

Values of n	1	2	3	4	5	6	7	8	9	10
Last digits for the powers given above										
2^n	2	4	8	6	2					
3^n	3	9	7	1	3					
$2^n + 3^n$	2^1+3^1 5	2^2+3^2 3	2^3+3^3							
7^n										
3^n+7^n	3^1+7^1 ?	3^2+7^2 ?								

NEXT

Can you make up an expression involving sums or products of powers of whole numbers and write down a list of the properties that you see. Perhaps exchange your work with a partner and check if you agree about the properties and that the properties are clearly described.

Notes for teachers

Solution

The table shows the last digits in the powers of 1, 2, 3, …9.

The last digits repeat in cycles.

The last digits of 2^n are 2, 4, 8, 6, 2, 4, 8, 6, … repeating the cycle 2, 4, 8, 6 over and over again
The last digits of 3^n are 3, 9, 7, 1, 3, 9, 7, 1 … repeating the cycle 3, 9, 7, 1 over and over again …
So adding $2^n + 3^n$ gives the last digit 5, 3, 5, 7, 5, 3, 5, 7 … repeating the cycle 5, 3, 5, 7 over and over again. The **numbers $2^n + 3^n$ are divisible by 5 for all odd values of n**.
The last digit is never zero so **$2^n + 3^n$ is never a power of 10 for any value of n**.

	n=1	n=2	n=3	n=4	n=5	n=6	n=7	n=8	n=9	n=10
	Last digits for n^{th} powers									
	1	1	1	1	1	1	1	1	1	1
	2	4	8	6	2	4	8	6	2	4
	3	9	7	1	3	9	7	1	3	9
2^n+3^n	5	3	5	7	5	3	5	7	5	3
	4	6	4	6	4	6	4	6	4	6
	5	5	5	5	5	5	5	5	5	5
	6	6	6	6	6	6	6	6	6	6
	7	9	3	1	7	9	3	1	7	9
3^n+7^n	0	8	0	2	0	8	0	2	0	8
	8	4	2	6	8	4	2	6	8	4
	9	1	9	1	9	1	9	1	9	1

The last digits in $3^1 + 7^1$, $3^3 + 7^3$, $3^5 + 7^5$, $3^7 + 7^7$, … and so on for all odd powers all end in 0 so these **numbers $3^n + 7^n$ are divisible by 10 (multiples of 10) for all odd values of n.**

Similarly the last digits in $1^3 + 9^3$, $1^5 + 9^5$, $1^7 + 9^7$, … and so on for all odd powers all end in 0 so these **numbers $1^n + 9^n$ are divisible by 10 (multiples of 10) for all odd values of n**.

Similarly the last digits in $2^3 + 8^3$, $2^5 + 8^5$, $2^7 + 8^7$, … and so on for all odd powers all end in 0 so these **numbers $2^n + 8^n$ are divisible by 10 (multiples of 10) for all odd values of n.**

Similarly the last digits in $4^3 + 6^3$, $4^5 + 6^5$, $4^7 + 6^7$, … and so on for all odd powers all end in 0 so these **numbers $4^n + 6^n$ are divisible by 10 (multiples of 10) for all odd values of n**.

Why do this activity

This activity provides practice for learners in working with exponents using both multiplication and addition, also practice in spotting number patterns and in mathematical reasoning and proof. It is the second in a set of similar activities that can be used in the same week or at different times, perhaps one each year to review and reinforce understanding of the operational rules and concepts relating to exponents.

This activity supports progress from the activity Powerful Thinking 1 which deals only in numbers as this one is written in algebraic terms expressing general results for all values of the variable n for example: $a^n + b^n$ is divisible by 5 for all odd values of n for all integers a and b such that (a + b) is divisible by 5 and $a^n + b^n$ is divisible by 10 for all odd values of n for all integers a and b such that (a + b) is divisible by 10. The open ended nature of the question allows learners to be creative in their thinking.

Diagnostic Assessment
This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Tom and Katie are arguing about $4^2 \times 2^2$

Tom says $4^2 \times 2^2 = 64$

Katie says $4^2 \times 2^2 = 8^4$

Who is correct?

A Only Tom

B Only Katie

C Both Tom and Katie

D Neither is correct

A is the correct answer

$4^2 \times 2^2 = 16 \times 4 = 64$ or $(4 \times 2)^2 = 8^2 = 64$

B. Katie's answer is wrong. She may be confused about adding powers when multiplying the same base number but in this example the powers are not added.

https://diagnosticquestions.com

Learning objectives
In doing this activity students will have an opportunity to:
- develop a deep understanding of exponents or powers;
- investigate some cyclic number patterns.

Generic competences
(some suggestions, select from list or write your own)

In doing this activity students will have an opportunity to **think flexibly**, be creative and innovative and apply knowledge and skills.

Suggestions for Teaching
This activity follows naturally from Powerful Thinking 1 https://aiminghigh.aimssec.ac.za/grades-8-10-powerful-thinking-1/ .

You could write the question on the board (or provide a worksheet) and introduce the table helping the learners to make a start filling in the table to record their findings. Then ask the learners to fill in the table on their own and then compare their results with a partner so that they can check each others' results and correct any mistakes. Then suggest that they answer the questions in pairs and then perhaps compare their findings with another pair.

Learners who have found all the pairs of integers between 1 and 10 that have powers adding up to a multiple of 10 can go on to creating their own number patterns

Finally ask the learners to explain how they have answered the question and ask them to give reasons for all their statements. Then give a summary of what they have learned about exponents making sure that all the language used is understood correctly.

You might ask where the learners have seen something like this before and connect it to division and recurring decimals, for example 2/3 = 0.666… with the digit 6 repeating again and again indefinitely and 5/7 = 0.7142857… with the digits 714285 repeating again and again indefinitely.

Key Questions

- What do you notice?
- What do you notice about the last digits?
- Can you explain how you know that?
- Can you describe a pattern that you see?
- Can you use algebra to make what you are saying into a general statement?

Follow up

See also Powerful Thinking 3 https://aiminghigh.aimssec.ac.za/grades-8-10-powerful-thinking-3/ and Powerful Thinking 4 https://aiminghigh.aimssec.ac.za/grades-10-powerful-thinking-4/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.
For resources for teaching A level mathematics see https://nrich.maths.org/12339

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6