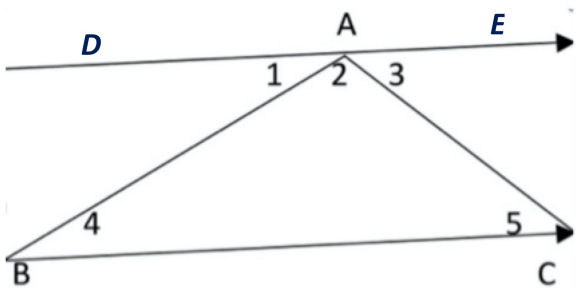
ANGLE SUM

For any triangle you can draw a line through one of the vertices **parallel to the opposite edge**.



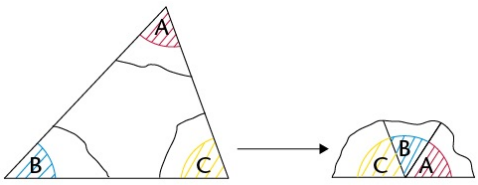
Here DE is parallel to BC.

What do you notice about the angles in the diagram?

What do you know about angles and parallel lines?

Can you use this diagram to prove that the angles of the triangle add up to 180 degrees?

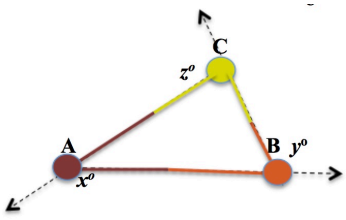
HELP



Draw triangles, carefully measure the angles and add them up. If you don't get a total even close to 180° then you need more practice in how to use a protractor, or you need to ask for help so that you can understand angles better.
Now cut out your triangles. Tear the corners off. Arrange the three angles of the triangle together as shown. The angles appear to lie on a straight line, therefore adding up to 180°. This is NOT A PROOF but it is convincing evidence.

[NB To make it obvious which are the **three angles tear edges, don't cut them.**]

NEXT – People maths



Explore the path around a triangle. Three people A, B and C make a triangle holding their arms out and holding hands with each other with their heads at A, B and C. A fourth person walks around the triangle, making a turn to change direction at each corner and ending up facing the same way as at the start. What total turn did he or she make? Do this as a 'people maths' enactment if you can.

Start at A and face along CA as shown by the dotted arrowline, then turn through angle x^o at A to go along AB, turn through y^o at B to go along BC and finally turning through z^o at C to face the direction started in. The angles x, y and z are called the **exterior angles** of the triangle. What is the total turn $x + y + z$? Can you explain why the sum of the exterior angles (otherwise called the total turn) is 360° (4 right angles)?

Also imagine an insect crawling around the triangle and ask the same questions about changes of direction. Think about the insect, it will help you to re-capture this idea at a later date, and to remember that the exterior angles add up to 360°.

At each vertex there is an exterior and an interior angle adding up to 180° (2 right angles).
What is the sum of **all** the exterior plus **all** the interior angles?
You know the sum of the exterior angles is 360° (4 right angles). What is the sum of the interior angles?
As a further extension you could do the same people maths enactment with 4 people for a quadrilateral, five for a pentagon etc. etc. … and use the same method to find the sum of the interior angles of any polygon.

NOTES FOR TEACHERS

SOLUTION

Line DAE is drawn through vertex A parallel to BC.

$\angle 4 = \angle 1$ (alternate angles DAE // BC)

$\angle 5 = \angle 3$ (alternate angles DAE // BC)

$\angle 1 + \angle 2 + \angle 3 = 180^o$ (angles on a straight line)

$\angle ABC + \angle BAC + \angle ACB = 180^o$ (equal to $\angle 1 + \angle 2 + \angle 3$)

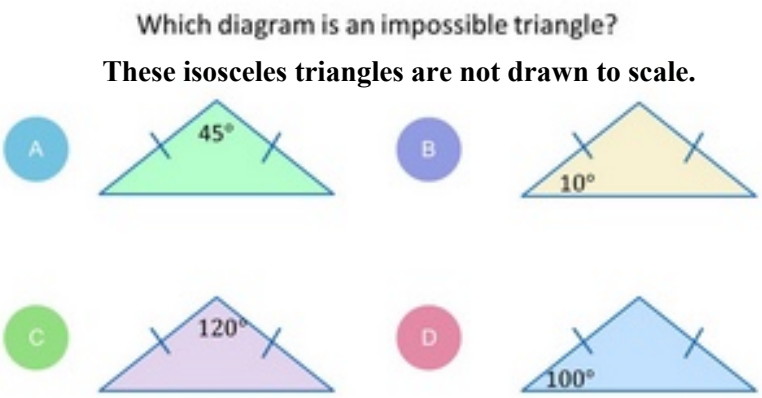
So the sum of the angles of ΔABC is 180^o.

Diagnostic Assessment

This should take about 5–10 minutes at the end of the lesson.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



1.Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2.Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

3.Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

The correct answer is **D** :

A. This triangle has angles 67.5^o, 67.5^o and 45^o.

B. This triangle has angles 10^o, 10^o and 160^o.

C. This triangle has angles 30^o, 30^o and 120^o.

https://diagnosticquestions.com

Why do this activity?

This activity provides a visual prompt for a method of proof that the angles of a triangle add up to 180^o suitable for learners in lower secondary school. This proof only requires a little simple reasoning and the knowledge that alternate angles with respect to parallel lines are equal and the angles on a straight line add up to 180^o. After learners have worked from it, then this diagram could provide a reminder of the proof and should be drawn in the learners' notebooks and shown on a poster on the classroom wall.

Learning objectives

In doing this activity students will have an opportunity to:

- learn to write a rigorous mathematical proof;
- review what they know about alternate, corresponding and co-interior angles and parallel lines

Generic competences

We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in new ways to produce solve today's complex problems and to improve the quality of life for all.

In doing this activity students will have an opportunity to:

- engage in independent learning;
- reason logically, apply knowledge and skills explain and record a rigorous proof.

Suggestions for teaching

Ideally learners will do this activity **before being taught** that the angles of a triangle add up to 180°. You could ask the learners to draw their own triangles, to measure the angles carefully and to add up the sum of the three angles. Then ask the totals and make a list on the board. Most will be between 175° and 185° and you would not expect many to be exactly 180° unless the learners already know what the sum should be.

Draw the diagram on the board or give copies to the learners. Tell the learners that **you want them to find out what the angles should add up to and if this will always be the same answer.**

If the learners already know the result then tell them that **you want them to prove** that the angles of a triangle add up to 180° **so they cannot assume it is true**. Ask the learners to write down all they know about the angles and to try to decide if this is enough to prove the result they want.

After the learners have worked on this activity the teacher can conduct a question and answer session **building on what the learners have done for themselves** so that the proof comes from what the learners themselves say, not from the teacher.

Finally, the teacher could require that all learners have a correct complete formal proof written in their notebooks.

Finish the lesson with the diagnostic quiz.

Key questions

- Can you see any angles in the diagram that are equal to each other? Why are they equal?
- What do you know about the angles on a straight line?
- What does that tell you about the angles of your triangle?
- Does the same method work for all triangles? Why?

Follow up

Carry out the PEOPLE MATHS enactment as described in the NEXT section. As a further extension do the same people maths enactment with 4 people for a quadrilateral, five for a pentagon etc. etc. … and use the same method to find the sum of the interior angles of any polygon.

For another activity see https://aiminghigh.aimssec.ac.za/years-9-12-tri-fold/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6