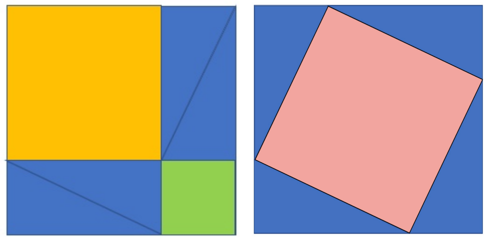
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

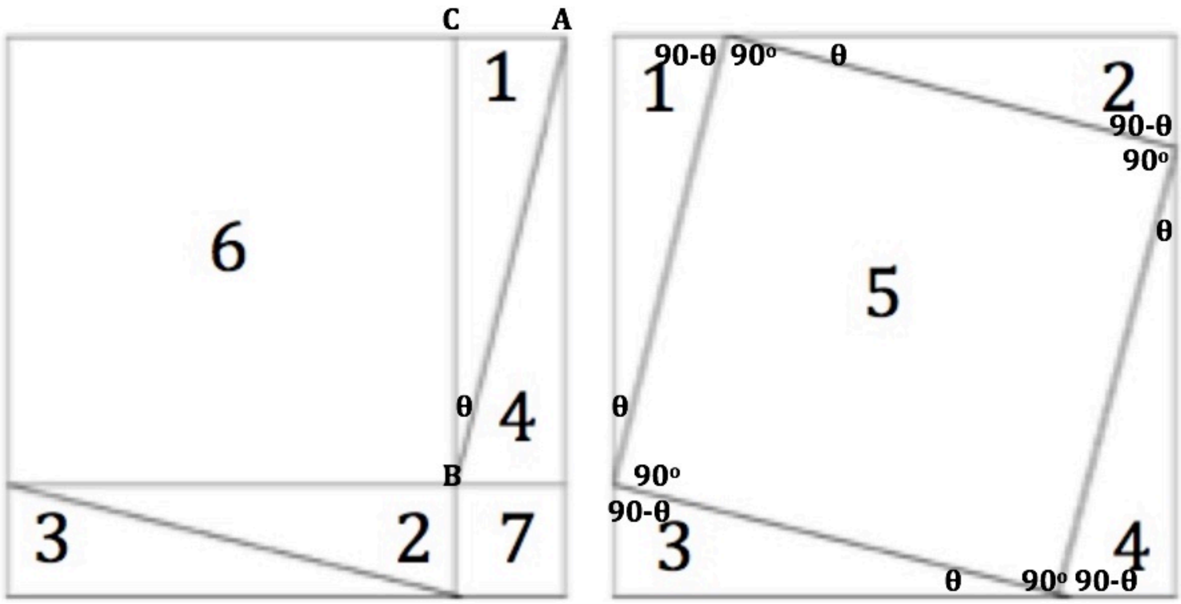
PYTHAGORAS JIGSAW

The diagrams show two different arrangements of pieces of the jigsaw inside a square frame. The right angled triangles numbered 1, 2, 3 and 4 are identical (congruent) copies of each other.

The Make Squares Jigsaw https://aiminghigh.aimssec.ac.za/years-6-12-make-squares-jigsaw/ has 7 pieces and these two solutions.



Here we refer to the edge lengths of the right angled triangle as *a* and *b* for the shorter edges and *c* for the hypotenuse. But *a,* b and *c* will have different values for each person who makes their own jigsaw like this choosing their own lengths. The outer frame is a square of edge length *a* + *b*.



Think about the area inside the outer square frame. How is that area made up in the two different arrangements? Just take away the four identical right angled triangles from each of the two arrangements. What is left?

What do you notice? What is the same? What is different? What can you deduce from the areas?

How can you be sure that the shapes labelled 5, 6 and 7 are squares?

What does this tell you about the area of square 5 compared with the areas of squares 6 and 7?

HELP

Cut out 4 congruent right angled triangles for yourself, choosing your own edge lengths, and arrange them in the first arrangement. Carefully draw the outline frame. Then arrange your 4 triangles in the frame in the second arrangement. Think about areas and answer the questions.

NEXT

1. Prove the algebraic identity $(a + b)^2 = a^2 + b^2 + 2ab$ using the areas in the diagram showing pieces 1, 2, 3, 4, 6 and 7.

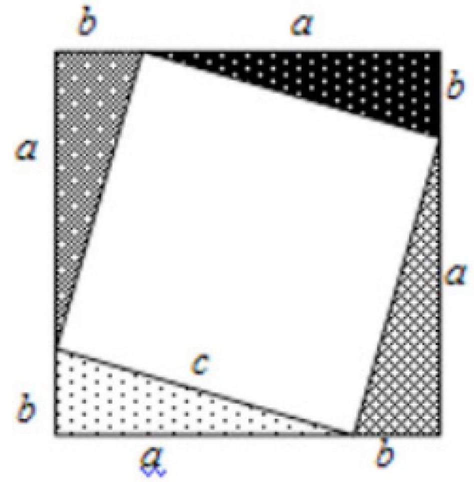
2. (a) Choose your own values of lengths *a* and *b*. Carefully and accurately draw a square of edge length $a + b$ measuring the lengths you have chosen and the angles of 90^o. Make 4 identical copies of your chosen right angled triangle. Cut out the 4 triangles and fit them in the square frame in the two different arrangements as shown in the diagrams. Measure length c and, using *a* calculator, check that $a^2 + b^2 = c^2$.

   (b) Why could it be that the calculation may not show this result exactly?

   (c) If you are working with a group or a whole class, make a table on the board with 5 columns headed *a, b, c*, $(a + b)^2$ and c^2. When you share these results what do you notice about columns 4 and 5? What does this tell you about all the right angled triangles that you have tested.

NOTES FOR TEACHERS

SOLUTION



The **4 congruent right angled triangles** numbered 1, 2, 3 and 4 in the diagrams all have edges of length *a, b* and c and angles 90^o, θ and 90-θ.

The 4 congruent triangles are arranged in the squares as shown in the diagrams.

The same diagrams can be drawn for any lengths *a* and *b*.

In shape 5 all the angles are 90^o because the angles on a straight line add up to 180^o.

So shape 5 is a square with all the edges of length *c* and area c^2; shape 6 is a square with edges of length *a* and area a^2; and shape 7 is a square with edges of length *b* and area b^2.

In one arrangement the shapes 1, 2, 3 4, 6 and 7 fill the same square frame measuring $(a + c)^2$.

In the other arrangement the shapes 1, 2, 3 4 and 5 fill the square frame measuring $(a + c)^2$.

The areas of shapes 6 and 7 in one arrangement and the area of shape 5 in the other arrangement take up the area: $(a + c)^2 - 2ab$ of the outer frame with 4 triangles removed.
This proves that: area of shapes 6 and 7: $(a^2 + b^2)$ = area of shape 5 (c^2).

That is the sum of the areas of the squares on the two shorter edges of a right angled triangle is equal to the area of the square on the hypotenuse.

This proves Pythagoras Theorem for all right angled triangles because the same argument works with the same diagrams for any values of *a* and *b*.

Note: there is no Diagnostic Assessment suggested here because the lesson 'flow' should not be interrupted from the time the learners engage in the lesson starter as they sit down in class. It might be useful for the teacher to use a diagnostic question in the previous lesson to review properties of congruent triangles.

Why do this activity?

It is not good teaching practice to introduce a result to learners without showing them why it is true. It is certainly bad practice to give one example (like the 3-4-5 triangle) and then to imply that the same result is true for all right angled triangles.

This activity provides a visual proof of Pythagoras Theorem that is suitable for learners in lower secondary school and can it be used when they are first introduced to Pythagoras Theorem.

This proof only requires a little simple reasoning and the knowledge that the angles of a triangle add up to 180^o, that angles on a straight line add up to 180^o and that the area of a square is equal to the square of the length of one edge.

Learning objectives

In doing this activity students will have an opportunity to experience thinking through a mathematical proof ***for themselves*** (guided re-invention) by using what they already know.

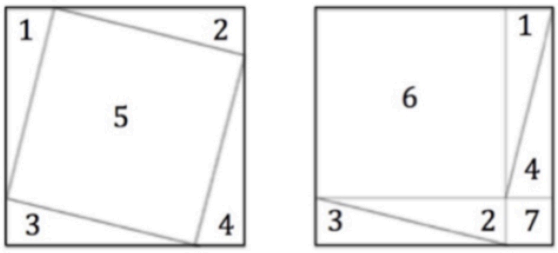
Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations and proofs;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **develop visualization** and skill to interpret or create images to represent concepts.

Suggestions for teaching

DO NOT TELL LEARNERS THAT THIS LESSON IS ABOUT PYTHAGORAS THEOREM.



What do you notice?

Lesson starter Draw the diagrams on the board, preferably before the learners arrive and settle down for the lesson. You could give copies to the learners but that is not necessary. Tell them to look at the diagrams and make a list of what is the same in the two big squares and what is different. They should do this individually for a few minutes and then compare their lists with a partner.

Then ask learners what they have noticed and make a list on the board.

Main lesson: Give the learners a piece of newspaper and ask them to fold it so they can cut out 4 congruent right angled triangles from 4 layers of paper. The triangles must be right angled but can have any other angles. Ask the learners to arrange their own triangles in the two layouts shown in the diagram.

Tell the learners that all the right angled triangles are congruent to triangle ABC and that angle ABC = θ.

Ask the learners to label all the angles in the diagrams and then to answer these questions about which there should be a class discussion.

- Think about the areas in the diagrams.
- Just take away the four identical right angled triangles from each of the diagrams. What is left?
- How can you be sure that the pieces labelled 5, 6 and 7 are squares?
- What does this tell you about the area of square 5 compared with the areas of squares 6 and 7?

Ending the lesson: After the learners have worked on this activity the teacher can draw out the necessary facts in a question and answer session **building on what the learners have done for themselves**.

Key questions

- Why is that angle a right angle (equal to 90^o)?
- Can you label all the right angles in the diagrams?

- Can you label all the angles equal θ, that is to angle ABC?
- What do you know about the sum of the angles in a triangle?
- What do you know about the angles on a straight line?
- What are the lengths of the edges of shape 5? Can you explain why you know it is a square?
- What are the lengths of the edges of shape 6? Can you explain why you know it is a square?
- What are the lengths of the edges of shape 7? Can you explain why you know it is a square?

Follow up

Make Squares Jigsaw (this would be good preparation before doing the Pythagoras Jigsaw activity): https://aiminghigh.aimssec.ac.za/years-6-12-make-squares-jigsaw/

Years 8 – 10 Riding on Pythagoras 1:
https://aiminghigh.aimssec.ac.za/years-8-10-riding-on-pythagoras-1/

Years 8 – 10 Riding on Pythagoras 2:
https://aiminghigh.aimssec.ac.za/years-8-10-riding-on-pythagoras-2/

Subscribe to MATHS TOYS if you have not already done so. https://www.youtube.com/mathstoys Or go to YouTube, search for Maths Toys.

Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and curriculum links: https://aiminghigh.aimssec.ac.za

Download the whole AIMSSEC collection of resources to use offline with the AIMSSEC App https://aimssec.app Or available to download from Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6