African Institute for Mathematical Sciences
Schools Enrichment Centre
AIMSSEC

MATHSLAND LOTTERY

In the Mathsland Lottery, 49 balls are numbered 1 to 49 and 6 balls are chosen at random without replacing any of the balls so that 6 different winning numbers are chosen.

Each lottery ticket has 6 numbers and you win a top prize if your 6 numbers match the 6 numbers chosen that week.

Is buying lottery tickets a waste of money? What is your chance of winning the top prize?

A good problem solving technique is to try simple cases if the problem seems difficult. The Lucky Numbers problem provides a simple case to try https://aiminghigh.aimssec.ac.za/grades-7-to-12-lucky-numbers/ .

You can also experiment by changing the numbers of balls in the interactivity on the NRICH website at: http://nrich.maths.org/7238

Solution

The probability that my first number matches is 6/49.
To win I must have 6 matches so next I work out the probability that both my first and my second numbers win, that is 6/49 x 5/48 (as we have already used one of the 49 numbers).
The probability of my first three numbers matching is 6/49 x 5/48 x 4/47
The probability of all my six numbers matching is
$6/49 \times 5/48 \times 4/47 \times 3/46 \times 2/45 \times 1/44 = 7.15 \times 10^{-8}$
or 1 in 13 983 816 million.

Notes for Teaching

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

A bag contains 4 blue counters and 2 yellow counters. The counters will not be replaced each time one is picked. What is the probability of getting two different coloured counters?

A $\frac{16}{30}$ B $\frac{8}{30}$ C $\frac{16}{60}$ D $\frac{10}{30}$

https://diagnosticquestions.com

5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

A. is the correct answer (4/6×2/5)+(2/6×4/5)

Common Misconceptions

B. Learners have only considered one of two possibilities.

C. or **D** Confused about probability or about fractions or just guessing

Why do this activity?

This activity offers an engaging context in which to develop students' understanding of experimental and theoretical probability. They can calculate theoretical probabilities, perhaps by first starting a tree diagram (my first number matches or my first number does not match) then moving towards multiplying fractions based on conditional probabilities. The activity also provides a basis for discussion about gambling in the light of it being a waste of money. Other aspects of gambling, such as addiction, can be discussed at the teacher's discretion.

Intended Learning Objectives

This activity will help learners to develop a deeper understanding of probability, especially of compound events.
It will help learners to relate what they learn in school to the real world and in this case to make them aware of the odds against the gambler.

Suggestions for Teaching

You could simulate the lottery in class by having 49 numbered cards in a bag. Each member of the class can choose 6 numbers. Then you draw 6 numbers from the bag. Does anyone in the class win? Does anyone have 5 matching numbers? Or 4?

Learners may not be able to start calculating the probabilities. Starting with simple cases is a good problem solving strategy. For a simple case, just put a few numbered cards in the bag (say 6) and draw out 2 cards. Ask learners to predict the chance of winning Then play the game. How many winners now? "Did we win as often as you expected? How could we calculate the probability of winning?" The class should now try to calculate the probability without the teacher suggesting how to do it. Give learners some time to work on this. Some students may list combinations (systematically or otherwise), whereas others may use tree diagrams. Move students on to consider the chances of winning with four balls (from six). Ideally, they will work on this using both listing and tree diagram approaches.

Bring the class together to share methods. Highlight anyone who has listed systematically to discuss the importance of making sure every combination is considered. Take time to discuss the symmetry that emerges from choosing a number on either side of three, and ask students to consider why this happens. If necessary, move students towards a tree diagram approach.

What if there were 20 cards and if players had to match 4 numbers? Ask learners to predict the probability of winning. Do a class experiment again. Then learners should calculate the probability of winning.

Working on this should give them enough confidence using tree diagrams to be able to solve the original problem.

Key questions

Did we win as often as you expected?

How could we calculate the probability of winning?

In the simple case of a lottery with 6 balls rather than 49, why is the probability of winning the two numbers from six lottery the same as the probability of winning the four numbers from six lottery?

Possible extension

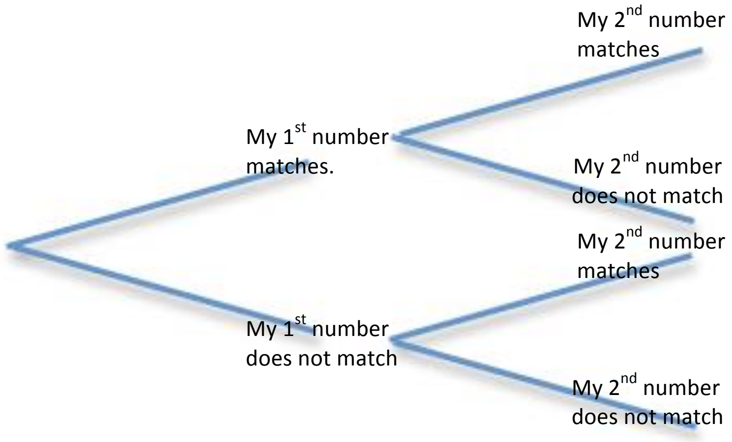
Students can be directed to http://en.wikipedia.org/wiki/South_African_National_Lottery to read about the South African National Lottery. The class can discuss the good and bad aspects of having a National Lottery, including addiction to gambling.

Students could consider the probability of matching 3, 4 or 5 numbers with the six numbers drawn.

Assuming that they would need to buy 7 million tickets in order to have a better than even chance of winning and they buy one ticket each week from the age of 18, and live for 98 years, students should calculate how many lifetimes they would have to go on buying tickets for in order to have a better than even chance of winning? (The answer is 1683 lifetimes).

Possible support

If the weakest learners are really struggling you could give them the tree diagram in this form and ask them to work in pairs or small groups to complete it.



Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. **Note:** The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.				
	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**