

LETTER LAND PUZZLE

The letters A, B, C, D, E, F, G and H correspond to the numbers from 0 to 7 with no repetitions.

Use the clues that $A+C=A$ and $F \times D=F$ to find the values of two letters.

Use the clues $B-G=G$ and $B/H=G$ to find the value of another three letters.

Use the clues $A+H=E$ and $E-G=F$ to find the remaining values?

Write the numbers corresponding to each letter as you work them out.

A B C D E F G H

Solution:

As $A + C = A$ we know $C = 0$

As $F \times D = F$ we know $D = 1$

As $B - G = G$ we know $B = 2G$ (double G)

Also $B/H = G$ means B divided by H = G so $B = H \times G$.

But we know $B = 2G$ so H must be 2.

As $B = 2G$, and the values 1 or 2 are already used, we know $B = 6$ and $G = 3$.

So we have to find A, E and F and the only remaining values are 4, 5 and 7.

As $A + H = E$ (with $H = 2$) we know $A = 5$ and $E = 7$ so F must be 4.

Using the clue $E - G = F$ we can check that the values $E = 7$, $G = 3$ and $F = 4$ are correct.

NOTES FOR TEACHERS

Diagnostic Assessment This should take about 5-10 minutes.

- Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
- Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
- If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

$$\begin{array}{r} 3 \text{ stars} = 36 \\ \triangle + \text{star} = 40 \end{array}$$

What is the value of the red triangle?



6



34



12



28

D. is the correct answer. We have $3 \times \text{star} = 36$, so $\text{star} = 36/3 = 12$. Then we have $\text{triangle} + 12 = 40$, so the triangle = 28.

Common Misconceptions

A. The student has identified the wrong variable, and has misread the star as a digit.

B. The student has misread the star as a digit, rather than a variable in a product.

C. The student has found the value of the star.

<https://diagnosticquestions.com>

Why do this activity?

The solutions to this problem can be found simply by mathematical (logical) thinking. This activity encourages learners to use the equations as clues rather than just applying rules, and to try to interpret what the equations signify about the relationships between the variables. The word 'puzzle' in the title also suggests this.

The activity also requires the learners to think about the connections between division and multiplication ($B/H=G$ is equivalent to $B=H \times G$) and between subtraction and addition ($B-G=G$ is equivalent to $B=G+G=2G$).

This non-routine question provides a bridge between the 'put the correct number in the empty box' type of exercise commonly used in primary schools, to the use of algebraic notation, and the development of skills in interpreting, re-arranging and solving equations that are vital to progress in mathematics.

Intended learning outcomes

Development of mathematical thinking.

Development of confidence in manipulating and interpreting algebraic equations.

Learners will think for themselves rather than following a method that has been shown to them.

Suggestions for teaching

The question is set in such a way as to suggest a path to finding the solutions. If you want to provide more challenge you can simply list the six clues. Then you can give the hints suggested one by one to learners as and when they need some help.

You may have to remind the learners that they should read the question carefully and use the fact that the solutions are given by the numbers 0 to 7 without repetition.

You might like to ask each group of learners to make a poster to explain how they found their solutions.

Or you may ask a pair of learners to come to the board and explain their method to the class.

After they have successfully found the solutions be sure to tell the learners that they have solved a set of equations with 8 unknowns and that these are called simultaneous equations. Then congratulate the learners and tell them that they only need to solve sets of equations with 2 unknowns in the school exams.

Key questions

Have you used all the facts that you are given?

Can you explain how you know that?

Have you checked your answers?

Which letters have you still to find?

Which numbers are not used yet?

Possible extension

Try the problem Shape Times Shape <http://nrich.maths.org/5714>

Possible Support

$$\text{■} + \text{■} = 5$$

$$\text{●} + \text{■} = 11$$

$$\text{■} + \text{■} = 8$$

In order to introduce the idea of a symbol representing a number, you could ask what numbers the shapes here would represent if they are all positive whole numbers.

Ask pairs of learners to talk to each other about how they can work out what each shape stands for in these calculations. In this case the last sum is actually the most helpful to start with.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.

Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not included in the school curriculum for Grade 12 SA.**

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6