HANDSHAKES

Everyone in the class must stand up and shake hands with everyone else and say hello. Everyone must join in. How many handshakes will there be altogether?

This will be difficult to count, especially if you have a large class. Try the problem in small groups and then compare different methods for finding the answers, or try it with a few learners at the front of the class and the remainder of the class counting the handshakes.

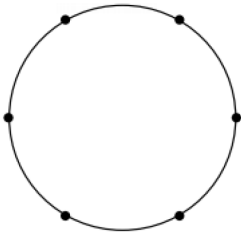
How many handshakes are there for 2 people, how many for 3 people, how many for 4 people and so on?

When the handshaking is finished discuss how many handshakes took place.

Can you see a pattern? Can you explain it? Can you find a method for getting the answer whatever the size of the group? Can you find different methods of finding the number of handshakes?

Help

When a problem seems hard to solve a good strategy is to try simple cases of the same problem.
How many handshakes would there be if there were only 2 students in the class?
How many for 3 students? How many for 4 students? How many for 5 students?



You could represent people shaking hands by joining points around a circle (Mystic Rose).
Try this for 6 people represented by 6 dots on a circle.
Can you see a pattern? Can you explain why the pattern occurs?

Extension

If everyone in a group shakes everyone's hand could the number of handshakes be 9, 19, 29, 39, ...?
Are these impossible? How do you know?

Work out whether the following numbers could be the number of handshakes at a mathematical meeting. If so how many people would there be at the meeting?

- 4851
- 6214
- 3655
- 7626
- 8656

What other impossible patterns can you find for numbers of handshakes?
Could there ever be a meeting with a multiple of 1000 handshakes? Explain your answer?

NOTES FOR TEACHERS

SOLUTION

Method 1 Everyone shakes hand with everyone else when, if there are 50 learners in the class, every learner must shake the hands of 49 other learners. But each handshake is counted twice. So there are altogether (50 x 49)/2 handshakes, that is 1225 handshakes.

Method 2 Everyone shakes hand with everyone else if the first learner shakes 49 hands then sits out, and then the second learner shakes 48 hands and sits out, and then the third learner shakes 47 hands and sits out and so on until there are only 2 learners left standing, and they shake hands and sit down.
This shows that the number of handshakes is
$49 + 48 + 47 + 46 + \ldots + 1$.
This is the triangle number T_{49} showing that $T_{49} = 49 + 48 + 47 + 46 + \ldots + 1 = (50 \times 49)/2$

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

James buys some oranges, then eats a few.
What would be the best start to forming an expression for the number of oranges he has left?

A Let x be the number of oranges James buys.
Let y be the number of oranges he has left.

B Let x be the price of the oranges.
Let y be the number of oranges James eats.

C Let x be the weight of each orange in grams.
Let y be the number of oranges James buys.

D Let x be the number of oranges James buys.
Let y be the number of oranges he eats.

D. **is the correct answer.**

Common Misconceptions

A. You need to know how many oranges he eats, in order to work out how many are left. Also, because you are looking for an expression, not an equation, you don't need a variable for the number of oranges left.

B. The price of the oranges has nothing to do with the question

C. The weight of the oranges has nothing to do with the question.

D. These two things will allow you to work out how many oranges he has left.
https://diagnosticquestions.com

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Why do this activity?

This activity offers learners an opportunity to relate numerical ideas to real life contexts. Thinking about different ways of counting the number of handshakes can lead to a better understanding of the general formula for adding the natural numbers $1 + 2 + 3 + 4 + \ldots + (n - 1)$ and the connection with triangle numbers.

It is clearly best to start with the special cases of 2 or 3 or 4 people, that is to simplify the problem in order to understand better how to solve it in general. Teachers should give learners a little time to decide this for themselves before you suggest it.

Intended learning objectives

Upper Primary
In doing this activity students will have an opportunity to:
- describe observed relationships or rules in learners' own words;
- investigate and extend numeric and geometric patterns looking for relationships or rules of patterns;
- solve problems involving whole numbers and number patterns.

Lower Secondary
- investigate and extend numeric and geometric patterns looking for relationships between numbers;
- describe and justify the general rules for observed relationships between numbers in learner's own words or in algebraic language.

Upper Secondary
- investigate number patterns leading to arithmetic sequences and summing series.

Generic competences

In doing this activity students will have an opportunity to:
- think critically/mathematically;
- reason logically – to be creative and innovative - to apply knowledge and skills;
- solve problems – to solve and interpret problems.

Suggestions for Teaching

You might ask the learners to work on the problem in small groups. Some groups will naturally use Method 1 and some will naturally use Method 2. The teacher can then lead a class discussion in which the different groups explain their methods and the teacher provides a summary at the end of the lesson of what has been learned.

Alternatively, if group work is not practical as in a very large class, ask for seven volunteers to come and stand at the front of the class, and ask each volunteer to shake hands with everyone else, with the rest of the class counting how many handshakes take place. Was it easy to count? Would it be useful for the volunteers to shake hands in a more systematic manner? Repeat the process in the way suggested in Method 2 above.

Allow some time for learners to work out how many handshakes there would be with 8, 9 and 10 people, and discuss answers and methods.

For a class that has been introduced to algebra, this can be generalised to give the formula for the nth triangle number, that is the sum of the natural numbers 1 to n.

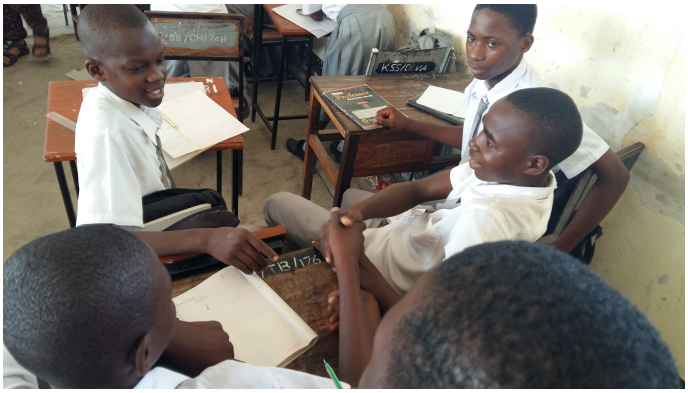
This activity works very well in conjunction with Triangle Number Picture and Mystic Rose. You could use the activities in successive years and remind the learners of previous work, discussing how the three examples illustrate the same mathematical concept. Or the whole class could work on all three problems, or small groups could be allocated one of the three problems to work on, and then report back to the rest of the class.

Key questions

- How can you be sure everyone has shaken hands with everyone else once and only once?
- How many handshakes would there be between 3 people? or 4 people?
- Can you use the same reasoning to find the answer for the whole class?
- Could you find the number of handshakes for the whole school without actually counting them?
- Can you explain how you found your answer?
- What is special about the numbers of handshakes in different sized classes?

Follow up

See: Triangle Number Picture https://aiminghigh.aimssec.ac.za/years-9-to-12-triangle-number-picture/
and Mystic Rose https://aiminghigh.aimssec.ac.za/years-7-12-mystic-rose/



Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**