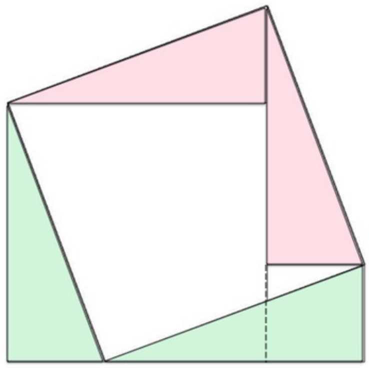
PYTHAGORAS SIMILARLY



There are hundreds of proofs of Pythagoras Theorem.

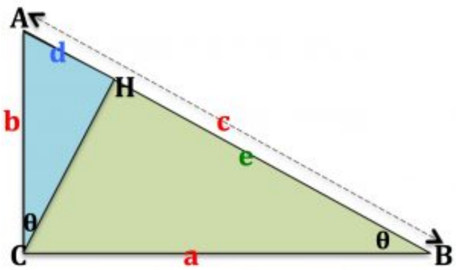
This diagram shows a proof.

Look carefully. Can you see three squares?

Assuming that they are squares, can you prove that four of the triangles are congruent?

Can you now write down a proof of Pythagoras' Theorem and explain the proof?

The second diagram shows a well known proof using similar triangles.



Angle ACB is a right angle and CH is perpendicular to AB.

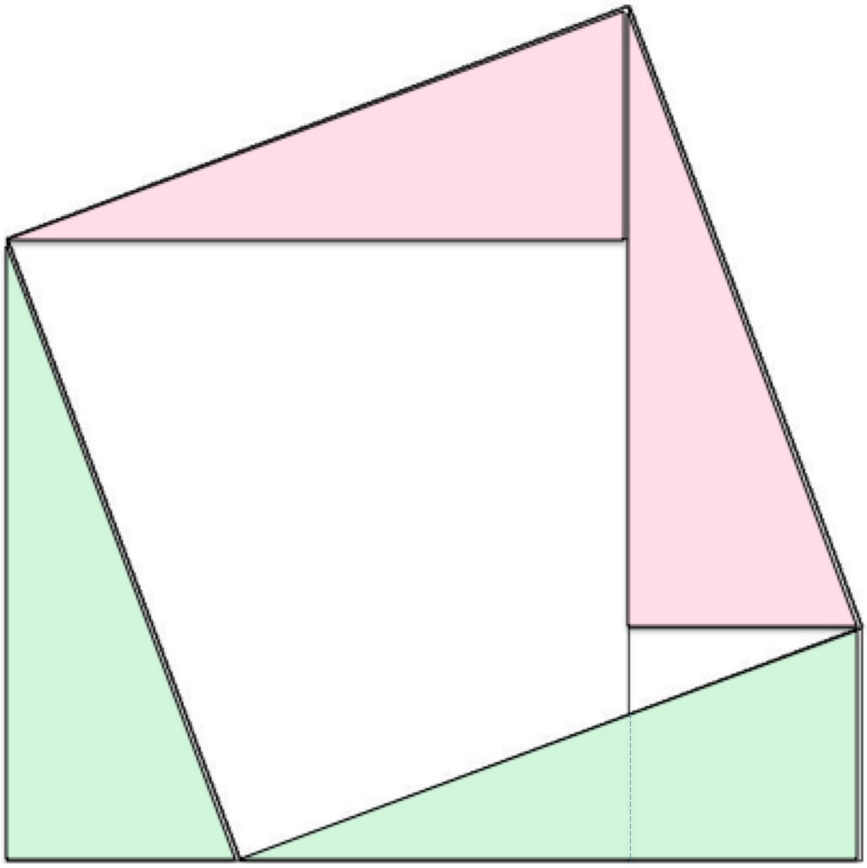
Explain why the two angles labelled θ must be equal.

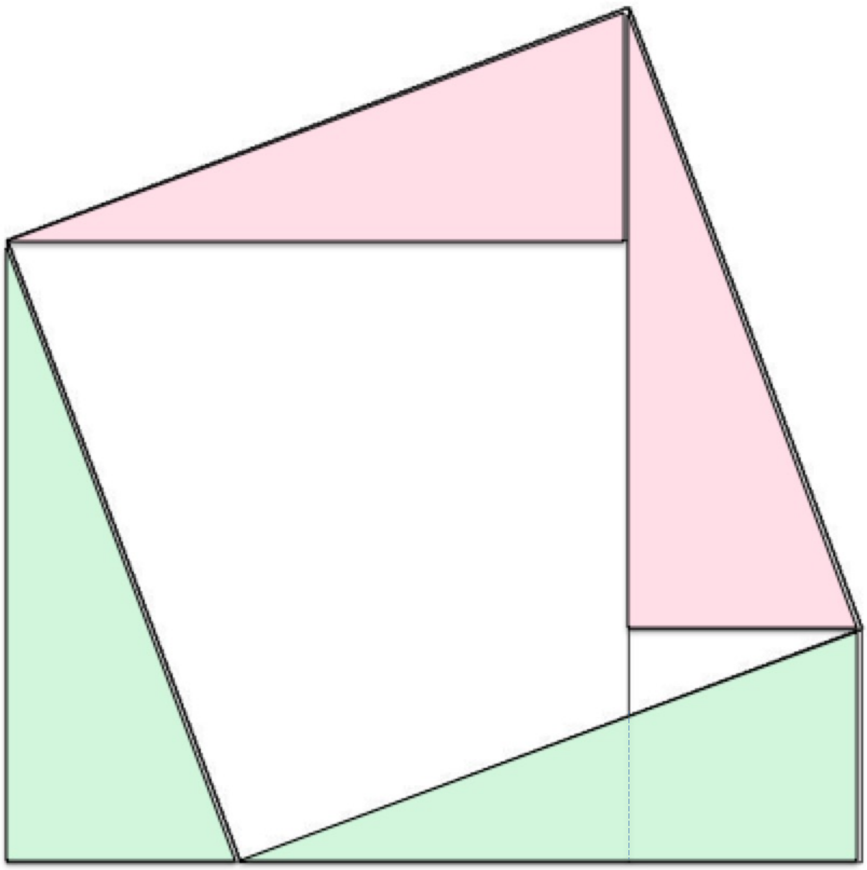
Explain why $b^2 = dc$ (1).

Explain why $a^2 = ec$ (2).

Combine equations (1) and (2) to give a proof of Pythagoras Theorem.

HELP





JIGSAW 1

Make 2 copies.

Cut each copy into 6 pieces.

Compare the triangles. What do you notice?

Put the pink triangles and the white pieces together to make a square.

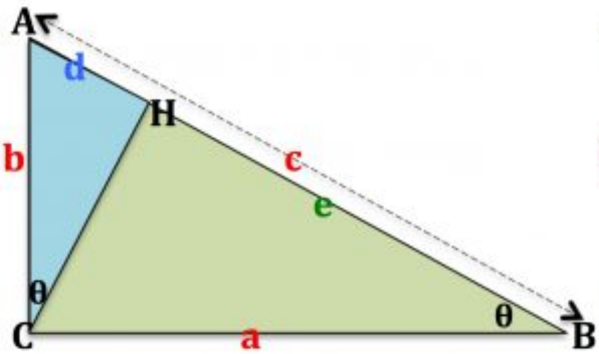
What do you notice about the square?

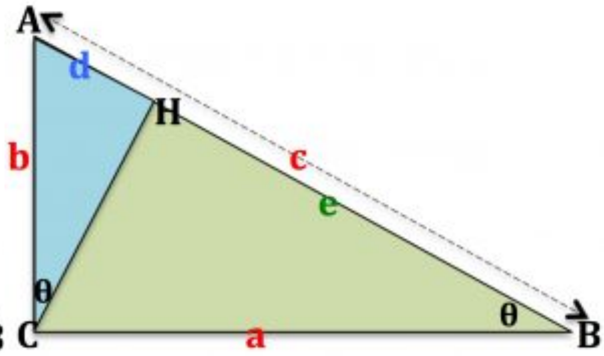
Put the green triangles and the white pieces together to make 2 squares.

What do you notice about the square?

What can you deduce from this activity?

PROOF BY SIMILAR TRIANGLES





Cut out 2 copies of the diagram. Cut one to make 2 triangles.

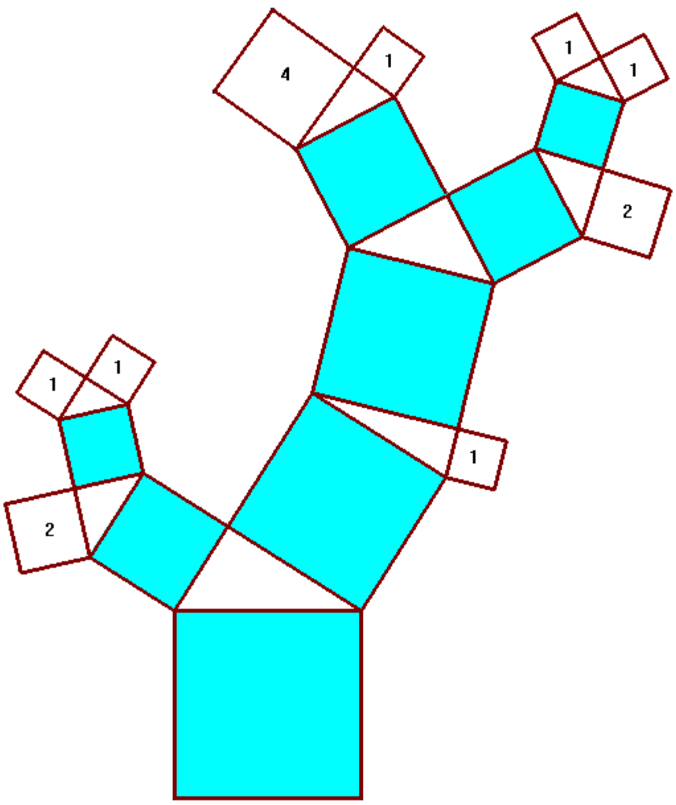
Place the 3 triangles to show how the angles correspond to the angles in triangle ABC showing that the three triangles are similar.

Triangle ABC is an enlargement of each of the smaller triangles. By what scale factors?

What can you deduce about lengths d and e?

What can you deduce by adding d and e?

NEXT



PYTHAGOREAN TREE

All the triangles in the tree are right-angled.

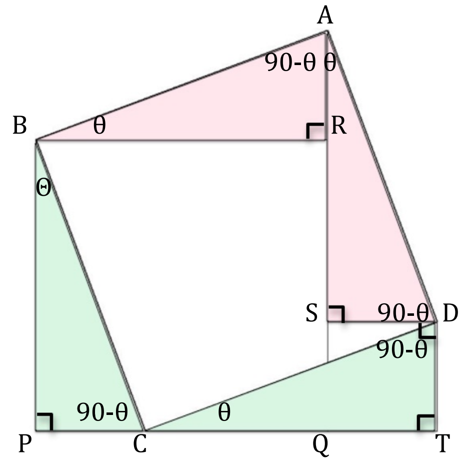
Cut out the tree into 24 pieces. Can you re-build the tree?

Work out the areas of the blue squares.

Draw some of your own Pythagorean Trees.

NOTES FOR TEACHERS

Solution



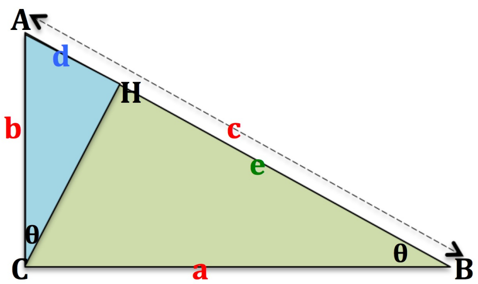
As ABCD, RBPQ and DSQT are squares it follows that triangles ABR, DAS, DCT and CBP have corresponding angles equal to $90^o - \theta$, θ and 90^o and the hypotenuses are equal in length.

So triangles ABR, DAS, DCT and CBP are congruent.

As congruent triangles are equal in area:

1. removing the two pink triangles ABR and DAS leaves the squares on the two shorter sides PQ and QT;
2. removing the two green triangles DCT and CBP leaves the square on the hypotenuse.

So the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the two shorter sides. This proves Pythagoras Theorem.



Angle ACB is a right angle and CH is perpendicular to AB.

If $\angle HBC=\theta$ then $\angle BCH=90-\theta$ because $\angle CHB=90^o$.

It follows that $\angle HCA=\theta$ because $\angle ACB=90^o$.

So triangles ABC, CBH and ACH are similar (equal angles).

So from triangles ACH and ABC we get $d/b = b/c$

and hence $b^2 = dc$ (1).

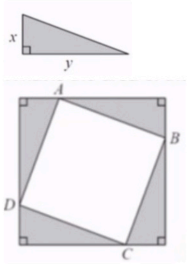
From triangles CBH and ABC we get $e/a = a/c$ and hence $a^2 = ec$ (2).

Combining equations (1) and (2) $a^2 + b^2 = ec + dc = (e + d)c = c^2$

which proves Pythagoras Theorem.

DIAGNOSTIC ASSESSMENT FOR SECONDARY STUDENTS

It does not require any knowledge of Pythagoras Theorem.
The assessment can be done as a class as described below or individually.
Show this question and say:
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



Four of these right-angled triangles surround the square ABCD just touching each other at these points.

What is the area of the **outer** square?

A. $x^2 + y^2$

B. $\sqrt{(x^2 + y^2)}$

C. $x^2 + 2xy + y^2$

D. $x + y$

1.Notice how the learners respond. Ask them to explain why they gave their answer and DO NOT say whether it is right or wrong, simply thank the learner for the answer.
2.It is important for learners to explain the reason for their answer so that, by putting their thinking into words, they develop communication skills and gain better understanding.
3.With a group, make sure that other learners listen to these reasons and try to decide if their own answer was right or wrong.
4.Ask the learners to vote for the right answer again by putting up 1, 2, 3 or 4 fingers. Look for a change and who gave right and wrong answers.

C. is the correct answer.
NOTE: This provides a proof of Pythagoras Theorem.
Area or outer square = $(x + y)^2$
Area of 4 triangles = $4(½xy) = 2xy$
Area of square ABCD = Area outer sq. – Area 4 triangles
$= (x + y)^2 - 2xy = x^2 + y^2$
Square of hypotenuse AB = Area of square ABCD
$= x^2 + y$ = Sum of areas of squares on the other 2 edges.

Common Misconceptions
A. May have known the answer is $(x + y)^2$ but been unable to expand this formula.
B. Clearly confused and blindly using a formula linked to Pythagoras Theorem without any understanding that this is a length not an area.
D. Very confused about area, or guessing – again this is a length.
https://diagnosticquestions.com

Why do this activity?
This Pythagoras Similarly activity provides a sequence of simple steps relating to each diagram which can be followed to produce two different proofs of Pythagoras Theorem. Learners can be given one or both of these diagrams and asked to follow the steps. **After they have worked on this activity** the teacher can draw out the necessary facts in a question and answer session building on what the learners have done for themselves.

This **Jigsaw Inclusion and Home Learning Guide** provides a collection of simple jigsaw puzzles all in some way related to right-angled triangles and Pythagoras Theorem. Teachers can provide for the individual learning needs of every learner in the class by assigning the most suitable learning activity. Learners can do one of the jigsaw puzzles or they may try more than one. All the jigsaw puzzles, in different ways, demonstrate Pythagoras Theorem.

There should be no pressure on learners to write formal proofs until they are at the Upper Secondary stage, but all learners can be asked to look for patterns in the diagrams and to explain what they see. Younger learners can simply put the pieces together and talk about the shapes and their properties.

Learning objectives

In doing this activity students will have an opportunity to:

- enjoy solving simple jigsaw puzzles;
- develop a familiarity with properties that lead to congruency and similarity of polygons in diagrams that exhibit relationships between areas of squares on the edges of right angled triangles.

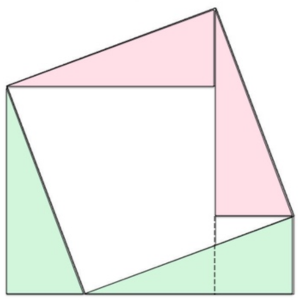
Generic competences

In doing this activity students will have an opportunity to:

- develop logical reasoning and problem solving skills
- develop powers of visualization;
- develop skills in explaining proofs.

Suggestions for teaching

First give the class a few minutes to decide on the answer to the diagnostic question. Take responses from the class and then discuss how you **don't need to USE Pythagoras Theorem** but the diagram suggests and illustrates **a proof** of the theorem.



There are many proofs of Pythagoras Theorem. You may choose to give one or both of these proofs according to the time available.

Proof 1 using congruent triangles

Give the first diagram and the questions with it to the learners. Label the points A, B, C, D, P, Q, R, S, and T. You may ask the learners to work on it individually for a few minutes to get started and then ask them to work in pairs.

If you can get around the classroom you can ask the key questions to pairs of learners according to their progress and whether you need to give more support to some individuals, perhaps giving them the HELP section (see page 2). If you cannot get around the classroom then write the key questions on the board and tell the learners the questions are there to help them.

You might ask pairs of learners to come to the board and explain their proof. Finally write the proof down on the board and explain again step by step and ask the learners to copy it into their notebooks.

Key questions – Proof 1

- How many right angled triangles can you see?
- Label all the angles that you know are equal to angle ABR (call them θ).
- Are any of the angles in the triangles equal to the angles in the other triangles? Why?
- Can you see any pairs of congruent triangles?
- What are you left with if you take away the two pink triangles?
- What are you left with if instead you take away the two green triangles?

- What does this tell you about the sum of the areas of the two smaller squares as compared to the area of the biggest square?

Proof 2 using similar triangles
Give the second diagram and the questions with it to the learners. You may ask the learners to work on it individually for a few minutes to get started and then ask them to work in pairs.

If you can get around the classroom you can ask the key questions to pairs of learners according to their progress and whether you need to give some individuals more support. If you cannot get around the classroom then write the key questions on the board and tell the learners the questions are there to help them.

You might ask pairs of learners to come to the board and explain their proof. Finally write the proof down on the board and explain again step by step and ask the learners to copy it into their notebooks.

Key questions – Proof 2
- Can you prove that all three triangles in the diagram are similar to each other?
- What do you know about the ratios of corresponding sides in similar triangles?
- What do lengths d and e add up to?

FOLLOW UP

See the Pythagorean Tree Activity on page 3

Riding on Pythagoras 1
https://aiminghigh.aimssec.ac.za/years-8-11-riding-on-pythagoras-1/

Riding on Pythagoras 2
https://aiminghigh.aimssec.ac.za/years-8-11-riding-on-pythagoras-2/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13