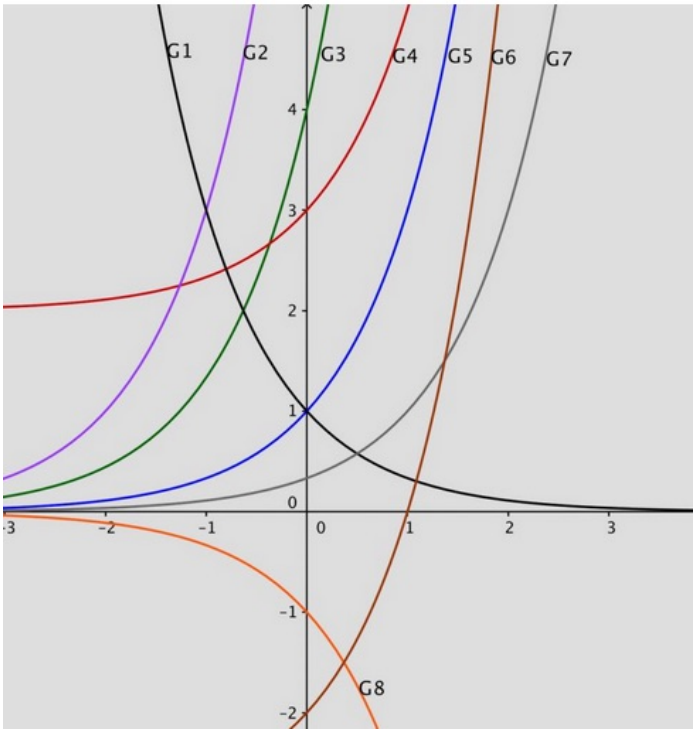
MATCHING EXPONENTIALS



1. Which one of the graphs represents the function $f(x)=3^x$?

Use your knowledge of transformations of other functions (e.g. lines, hyperbolas and parabolas) to match the graphs to their equations and to the descriptions of the transformations of $f(x)=3^x$.

Cut up a copy of the table on page 4 and rearrange the pieces into 8 matching sets.

Equations	Graphs	Transformations
1. $y = 4.3^x$	**G1**	**A.** $f(x)$ shifted 3 units downwards
2. $y = 3^x + 2$	**G2**	**B.** $f(x)$ reflected in the y-axis
3. $y = -3^x$	**G3**	**C.** $f(x)$ shifted 1 unit to the right
4. $y = 3^{x+2}$	**G4**	**D.** $f(x)$ shifted 2 units upwards
5. $y = 3^{x-1}$	**G5**	**E.** $f(x)$ reflected in the x-axis
6. $y = 3^{-x}$	**G6**	**F.** $f(x)$ shifted 2 units to the left
7. $y = 3^x - 3$	**G7**	**G.** $f(x)$
8. $y = 3^x$	**G8**	**H.** $f(x)$ intersects y-axis at (0 ; 4)

2. Given: $f(x) = 5^{-x}$

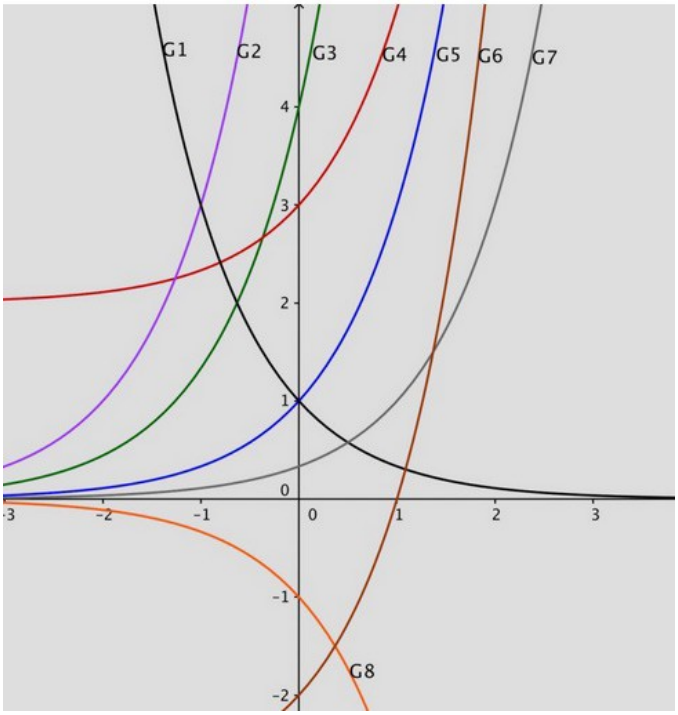
2.1 Write down the coordinates of A if A is the y-intercept of $f(x)$.

2.2 Write down the coordinates of the y-intercept of $g(x) = 2.5^{-x}$

2.3 Write down the coordinates of the y-intercept of $h(x) = 5^{-x} + 3$

2.4 Write down the coordinates of the y-intercept of $p(x) = 5^{-x-2} + 3$

2.5 Write down the coordinates of the y-intercept of $q(x) = -2.5^{-x} - 1$



1. Which one of the graphs represents the function $f(x)=3^x$?
Use your knowledge of transformations of other functions to match the graphs G1 to G8 to their equations numbered 1 to 8 and to the descriptions of the transformations of $f(x)=3^x$ labelled A to H.

Equation 1 has been done for you.

Equation	Graph	Description/Transformation
1. $y = 4.3^x$	G3	H

2. Given: $f(x) = 5^{-x}$

Write down the coordinates of A if A is the y-intercept of $f(x)$.	
Write down the coordinates of the y-intercept of $g(x) = 2.5^{-x}$	
Write down the coordinates of the y-intercept of $h(x) = 5^{-x} + 3$	
Write down the coordinates of the y-intercept of $p(x) = 5^{-x-2} + 3$	
Write down the coordinates of the y-intercept of $q(x) = -2.5^{-x} - 1$	

HELP

Start by putting $x = 0$ in the equations to find the y intercept and then identify the graph from the y intercept.

NEXT

Make up a similar activity of your own, perhaps with just 4 graphs and 12 cards altogether. Then exchange your work with another learner and each try to complete the matching activity created by the other learner, but also to spot any errors that they think have been made in creating the problem. The learners should try to resolve issues about errors and come up with a set of 24 cards between them that provide a similar activity to the original one.

Cut the table into 24 cards and rearrange them into 8 matching sets.

Equations	Graphs	Transformations
1. $y = 4.3^x$	**G1**	**A.** $f(x)$ shifted 3 units downwards
2. $y = 3^x + 2$	**G2**	**B.** $f(x)$ reflected in the y-axis
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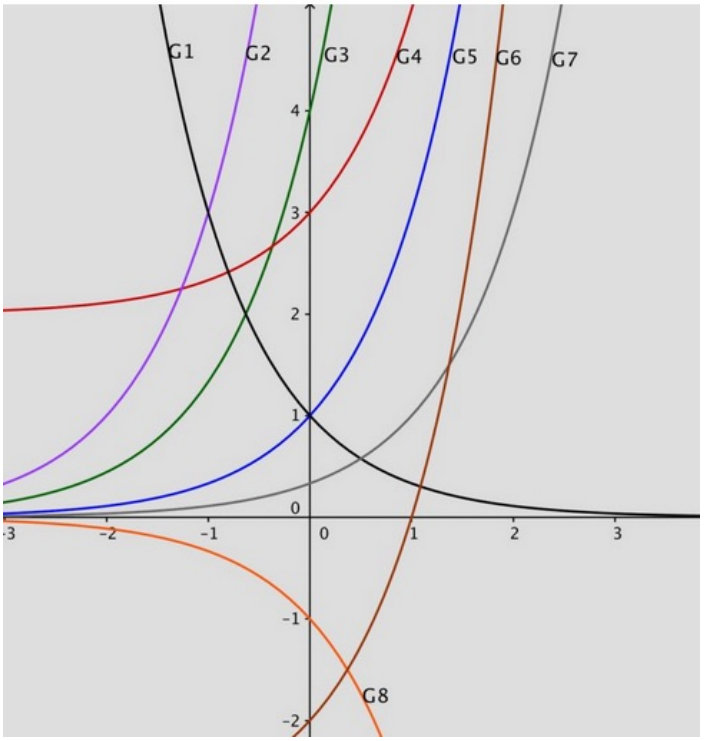
7. $y = 3^x - 3$	**G7**	**G.** $f(x)$
8. $y = 3^x$	**G8**	**H.** $f(x)$ intersects y-axis at (0 ; 4)

NOTES FOR TEACHERS

SOLUTION



1. Which one of the graphs represents the function $f(x)=3^x$?

Use your knowledge of transformations of other functions (e.g. lines, hyperbolas and parabolas) to match the graphs to their equations and to the descriptions of the transformations of $f(x)=3^x$.

Cut up a copy of the table below and rearrange the pieces into 8 matching sets.

SOLUTIONS

G5 is the graph of $f(x)=3^x$.

1: G3 $y = 4.3^x$ H
2: G4 $y = 3^x + 2$ D
3: G8 $y = -3^x$ E
4: G2 $y = 3^{x+2}$ F
5: G7 $y = 3^{x-1}$ C
6: G1 $y = 3^{-x}$ B
7: G6 $y = 3^x - 3$ A
8: G5 $y = 3^x$ G

	Equations	Graphs		Transformations
1	$y = 4.3^x$	**G1**	**A**	$f(x)$ shifted 3 units downwards
2	$y = 3^x + 2$	**G2**	**B**	$f(x)$ was reflected in the y −axis
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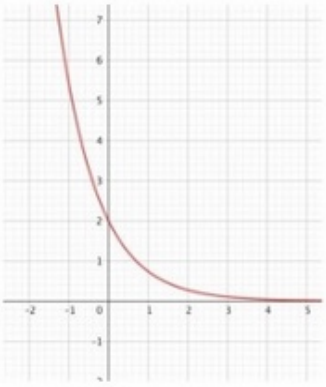
8	$y = 3^x$	**G8**	**H**	$f(x)$ intersects y-axis at (0; 4)

2. Given: $f(x) = 5^{-x}$ **SOLUTIONS**

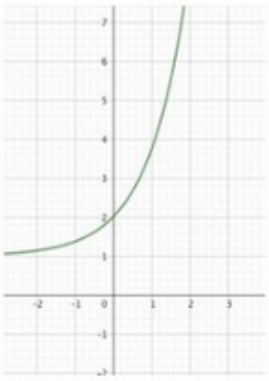
2.1 Write down the coordinates of A if A is the y-intercept of $f(x)$. (0 , 1)
2.2 Write down the coordinates of the y-intercept of $g(x) = 2.5^{-x}$ (0 , 2)
2.3 Write down the coordinates of the y-intercept of $h(x) = 5^{-x} + 3$ (0 , 3)
2.4 Write down the coordinates of the y-intercept of $p(x) = 5^{-x-2} + 3$ (0 , 3.04)
2.5 Write down the coordinates of the y-intercept of $q(x) = -2.5^{-x} - 1$ (0 , -3)

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



Graph A



Graph B

$f_1(x) = e^{-x} + 2$ $f_2(x) = -e^x + 1$ $f_3(x) = 2e^{-x}$ $f_4 = e^x + 1$

A graph A is $f_3(x)$ and graph B is $f_4(x)$

B graph A is $f_1(x)$ and graph B is $f_4(x)$

C graph A is $f_2(x)$ and graph B is $f_4(x)$

D graph A is $f_2(x)$ and graph B is $f_3(x)$

The correct answer is A

B. The mistake here is failure to recognise that the horizontal asymptote for graph A is y=0

C. The mistake here could be thinking the graph of e^{-x} was translated up one unit and failing to understand the $-e^{-x}$ takes negative values for all x.

D. The graph B cannot be f_3 because it has a positive gradient everywhere and f_3 has a negative gradient everywhere.

https://diagnosticquestions.com

Why do this activity?

This activity provides practice for learners in recognizing transformations of exponential functions and identifying the effects of transformations on the graphs of functions.

Learning objectives

In doing this activity students will have an opportunity to:

- gain a deeper understanding of exponential functions;
- gain a deeper understanding of transformations of graphs.

Generic competences (some suggestions, select from list or write your own)

In doing this activity students will have an opportunity to **persevere and work systematically** to investigate all possible cases.

Suggestions for teaching

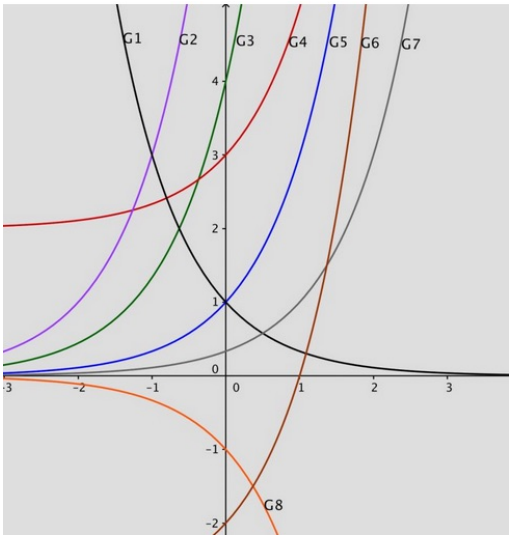
Start with the Diagnostic Quiz to review the exponential function.

Make copies of the table and give to the learners to cut out into 24 separate cards. Then they can work either in pairs or in small groups to sort and match the cards into sets.

Alternatively copy the cards in very large writing onto 24 separate sheets of A4 paper and stick them on the board in a random order. Ask the learners to work in pairs to match each equation with its graph and with the description of the transformation.

Those who finish first can carry on to answer question 2 or this could be a follow-up or homework task.

Then ask the learners to come up to the board one at a time to rearrange into the 8 sets the sheets that were stuck on the board.



Finally ask the learners that happens to the graph of $f(x) = e^x$ (graph G5) when you change the parameters in the equation $f(x) = ae^{k(x+p)} + q$ referring to the transformations in the graphs G1 to G7.

Key questions

- What is the y-intercept for that graph?
- How do you find the y-intercept from the equation?
- What is the y coordinate when x = 0 for that function?
- What transformation has changed that graph to that one?
- Does the graph show an increasing or decreasing function?
- What transformation of the function corresponds to that change in the equation?
- What change in the equation corresponds to that transformation of the function?

Follow up

See the similar questions on families of trig functions and quadratics:
Parabolic Patterns https://nrich.maths.org/parabolicpatterns
More Parabolic Patterns https://nrich.maths.org/777
Parabolas Again https://nrich.maths.org/791
Sine Problem https://nrich.maths.org/436
Tangled Trig Graphs https://aiminghigh.aimssec.ac.za/years-10-12-tangled-trig-graphs/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA. For resources for teaching A level mathematics see https://nrich.maths.org/12339				
	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6