HOW OLD AM I?

It is my birthday and in 15 years' time, my age will be the square of my age 15 years ago! Can you work out how old I am?

Now I am thinking "Was there ever a time in my life when I had other special birthdays?" Could I have said: "In 3 years' time, my age will be the square of my age 3 years ago" or: "In 4 years' time, my age will be the square of my age 4 years ago" or: "In 5 years' time, my age will be the square of my age 5 years ago" or...?

Can you make any generalisations about which birthdays are special in this way? Can you prove your findings?

HELP

1. What are you trying to find out? Let's use x for the unknown.
2. Can you write down an equation from the information given?
3. Can you solve your equation?
4. Check your solutions, are they both solutions to the problem?

NEXT

GENERALISATION: A good problem solving technique is to try simple cases. Are there any other special birthdays?

Try the same problem with 1, 2, 3, 4, 5 ... in place of 15.

You will discover that the special birthdays are 3, 6, 10, 15, 21, 28, 36, 45, ….

Do you recognise this sequence of numbers? This is the sequence of triangle numbers. Why do you think the solutions to this problem belong to this special sequence?

NOTES FOR TEACHERS

SOLUTION

If I am x years old then 15 years ago I was x - 15 years of age and in 15 years I will be x + 15.

From the information we can write down the equation $(x – 15)^2 = x + 15$ which simplifies to:

$x^2 – 30x + 225 = x + 15$ or

$x^2 - 31x + 210 = 0$ and factorising this equation we get

$(x – 10)(x – 21)$ which has solutions x = 10 and x = 21.

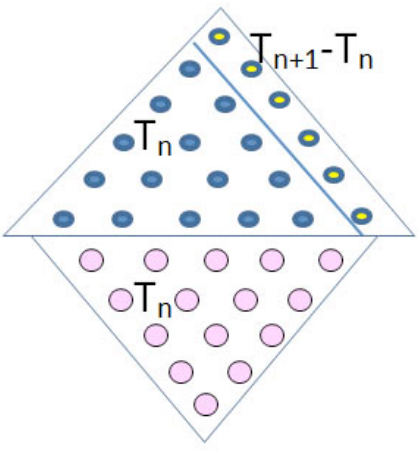
On this case $x \neq 10$ because we know $x > 15$ so the age we are looking for is 21.

Trying the same problem with 1, 2, 3, 4, 5 ... in place of 15 gives the special birthdays in the table below.

Time past and future : k	1	3	6	10	15	21	28	36	45		
My age now : x	3	6	10	15	21	28	36	45	55		
$(x – k)^2$	4	9	16	25	36	49	64	81	100		
$(x + k)$	4	9	16	25	36	49	64	81	100		

Notice that both the sequences of values of *x* and k are sequences of triangle number so a natural question to ask is:

“Once we find out that k is a triangle number could we immediately know (without doing the algebra) that my age now is a triangle number?” The answer is Yes, from the perfect symmetry in *x* and k of the equation $(x – k)^2 = (x + k)$ we know that if k is a triangle number then *x* must also be a triangle number.



$T_{n+1}+T_n = (T_{n+1}-T_n)^2$

We can see from this dotty pattern that the equation $(x – k)^2 = (x + k)$ must have solutions that are consecutive triangle numbers

The diagram shows for n = 5

$$\mathbf{T_{n+1} + T_n = [T_{n+1} – T_n]^2 = (n + 1)^2}$$

This formula can be proved algebraically for any two consecutive triangle numbers: ½n(n+1) and ½(n+1)(n+2) because they have the sum $(n + 1)^2$ and the difference (n+1)

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task. or give a remedial task.

What are the correct solution for the following pair of simultaneous equations:

$$y = x^2$$
$$y = x + 2$$

A $x = -1, y = 1$ *and* $x = 2, y = 4$

B $x = -1, y = 1$

C $x = -1$ *and* 2

D $x = 1, y=1$ and $x = 2, y = 4$

A. is the correct answer.

Common Misconceptions

B. The learner has only found one solution, There is another one where $x = 2$

C. The learner has calculated the values of x but not then found the values of y

D. The learner has written all values as positive, when one of the x values is negative ie -1

https://diagnosticquestions.com

Why do this activity?

This is an exercise on quadratic equations. It gives a simple context for learners to read and interpret text and to formulate and solve an equation. It also leads naturally to generalisations. The problem is useful for learners of all attainment levels. All learners who can solve quadratic equations should have success in finding a quadratic equation and solving it. There is a rich and surprising set of results to be discovered to challenge the high flyers, providing ideas to help them to develop mathematical thinking.

Learning objectives

In doing this activity students will have an opportunity to:

- get practice in solving quadratic equations;
- get practice in problem solving solving;
- respond to a challenge and get practice at carrying through an investigation, making generalisations, thinking mathematically and making and proving conjectures.

Generic competences (some suggestions, select from list or write your own)

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative, make and prove conjectures, and apply knowledge and skills;
- **persevere and work systematically** to investigate all possible cases.

Suggestions for teaching

Start with the diagnostic question. This will help learners to review what they know about quadratic equations.

Try this problem with your learners. It is a good problem for engaging learners in a mathematical investigation which links ideas from school mathematics and shows some of the power and beauty of mathematics (for example the use of a symmetry argument).

If the learners answer the Key Questions then they will have solutions to this problem and to the generalisations following it, with reasons for the pattern that occurs in the sequence of solutions.

Use the key questions to guide your learners in their thinking. Don't be tempted to tell them too much. Try to train your learners **to think for themselves**.

Key questions

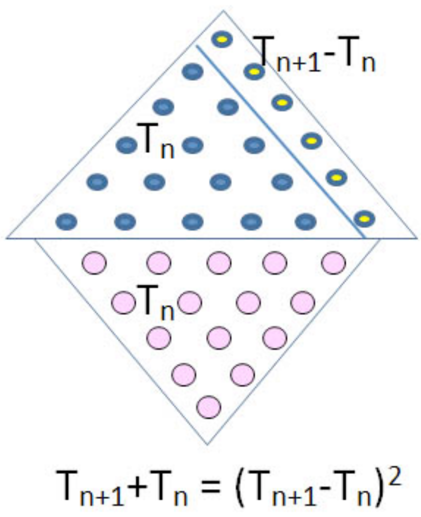
Remember a reason should be given for every answer.

1. What are you trying to find out? Let's use *x* for the unknown.
2. Can you write down an equation from the information given? [$(x – 15)^2 = x + 15$.]
3. Can you solve your equation? [x = 10 and 21]
4. Check your solutions, are they both solutions to the problem? [The only solution is x = 21 and not x = 10 because we know that x > 15.]

Good! Now you could stop here.

GENERALISATION: A good problem solving technique is to try simple cases. Try the same problem with 1, 2, 3, 4, 5 ... in place of 15. Are there any other special birthdays?

5. The special birthdays are 3, 6, 10, 15, 21, 28, 36, 45, …. Do you recognise this sequence of numbers? Why do you think the solutions to this problem belong to this special sequence? [This is the sequence of triangle numbers.]
6. Let y be my age k years ago then the equation becomes: $y^2 = y + 2k$ or $y^2 – y – 2k = 0$



How can you factorise the equation? For this equation to have whole number solutions the factors of 2k must differ by 1 (to get –y) so the equation factorises to:[y + n][y – (n+1)] = 0 so 2k = n(n+1) making k the triangle number ½n(n + 1).The solution (which must be positive) is y = n + 1 and my age now is y = (n + 1) + ½n(n + 1) = ½(n + 1)(n + 2) , the next triangle number.

7. Once we find out that k is a triangle number could we immediately know (without doing the algebra) that my age now is a triangle number? [Yes, from the perfect symmetry in x and k of the equation $(x – k)^2 = (x + k)$ we know that if k is a triangle number then x must also be a triangle number.]
8. What can you see from this picture?

Follow up

Handshakes https://aiminghigh.aimssec.ac.za/years-9-12-handshakes/
Mystic Rose https://aiminghigh.aimssec.ac.za/years-7-12-mystic-rose/
Triangle Number Picture https://aiminghigh.aimssec.ac.za/years-9-to-12-triangle-number-picture/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA. For resources for teaching A level mathematics see https://nrich.maths.org/12339				
	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6