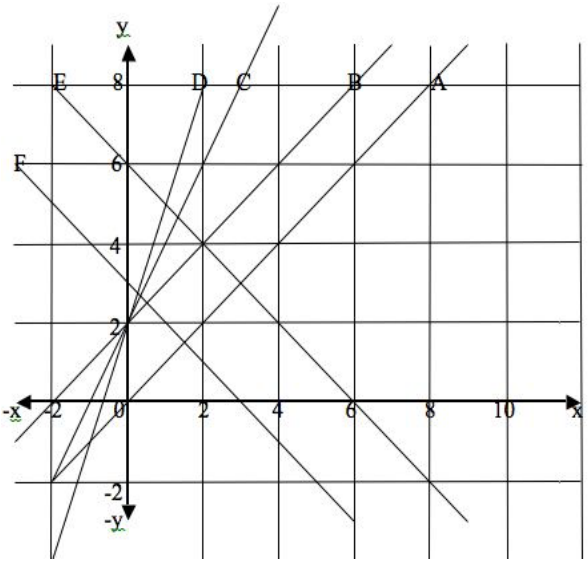
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

LINES



1. For each line fill in your answers to the following questions in the table below:
(Line E has been done for you.)
(i) Write down the coordinates (0; c) of the point of intersection of the line with the y-axis (the y intercept).

(ii) Choose two points on the line $(x_1 ; y_1)$, $(x_2 ; y_2)$. Find the gradient of the line from the formula $m = (y_2 - y_1) / (x_2 - x_1)$

(iii) Write the equation of the line you found in (ii) the form $y = mx + c$. What values correspond to m and c?

2. What do you notice when you compare the equations of these lines?

Line	Choose any 2 points on the line	Gradient Rise/Tread m	Intercept c	Equation of line
A	(;) (;)			
B	(;) (;)			
C	(;) (;)			
D	(;) (;)			
E	(- 2 ; 8) (2 ; 4)	Rise/Tread = -4/4 = -1		$y = - x + 6$
F	(;) (;)			

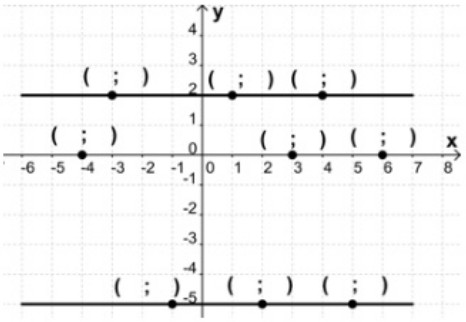
HELP

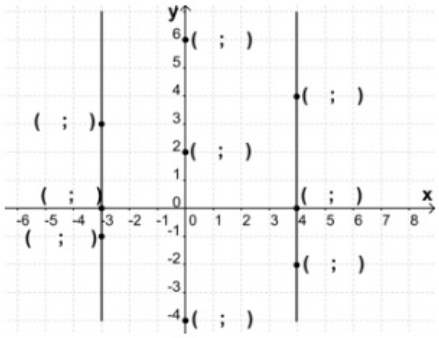
Think of the RISE as the distance UP between the points.
Think of the TREAD as the distance ACROSS between the points (like stairs).
Think of going to the right or up as positive changes.
Think of going to the left or down as negative changes.

Perhaps it would be a good idea to work with a partner so that you can help each other.

NEXT

1. Copy the graphs and fill in the coordinates of the points.





What are the gradients of the x-axis, the y-axis and the other 4 lines shown?

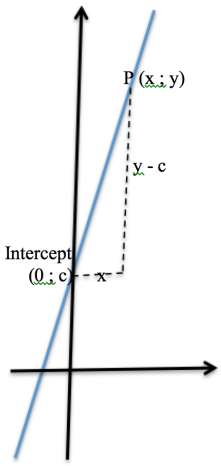
Write down the equations of the x-axis, the y-axis and the other 4 lines shown.

2. Match Column B to column A:

	Column A	Column B
2.1	Equation of the y-axis	$x = k$ (k is a real number)
2.2	Equation of a line parallel to the y-axis	$y = 0$
2.3	Equation of the x-axis	$y = c$ (c is a real number)
2.4	Equation of a line parallel to the x-axis	$x = 0$

HOME LEARNING GUIDE

SOLUTION



Equations of lines

The line joining 2 points $(x_1 ; y_1)$, $(x_2 ; y_2)$ has gradient $m = (y_2 - y_1) / (x_2 - x_1)$

If a straight line has gradient m

and intercept on the y-axis (0 ; c)

then for any general point on the line with coordinates (x ; y)

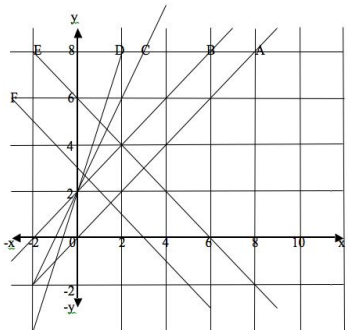
the gradient is given by

$$m = \frac{\text{rise}}{\text{tread}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y - c}{x}$$

Rearranging this equation gives the equation of the line in the form

$$\boxed{y = mx + c}$$

Fill in the table below from this diagram.



Line	Choose any 2 points on the line	Gradient Rise/Tread m	Intercept c	Equation of line
A	(8 ; 8) (0 ; 0)	8/8 = 1	0	$y = x$
B	(6 ; 8) (0 ; 2)	6/6 = 1	2	$y = x + 2$
C	(3 ; 8) (0 ; 2)	6/3 = 2	2	$y = 2x + 2$
D	(2 ; 8) (0 ; 2)	6/2 = 3	2	$y = 3x + 2$
E	(-2 ; 8) (2 ; 4)	Rise/Tread = -4/4 = -1	6	$y = -x + 6$
F	(-2 ; 5) (3 ; 0)	-5/5 = -1	3	$y = -x + 3$

The coefficient of x gives the gradient of the line.

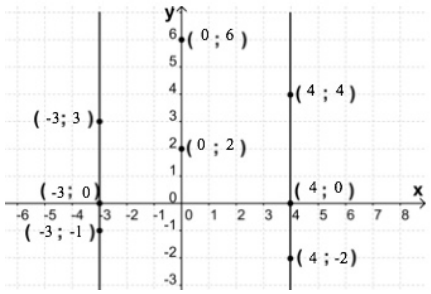
The constant term gives the y coordinate of the point where the line cuts the y-axis (the y-intercept).

Lines A and B are parallel and have equal gradients of +1.

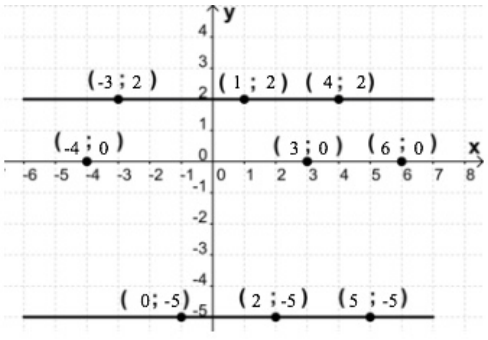
Lines E and F are parallel and have equal gradients of -1.

NEXT

1.



The lines are parallel to the y axis and have infinite gradient. The equation of the y-axis is $x = 0$ and the other lines have equations $x = -3$ and $x = 4$.



The lines are parallel to the x-axis and have gradient zero. The equation of the x-axis is $y = 0$ and the other lines have equations $y = 2$ and $y = -5$.

2.Match Column B to column A:

Column A		Column B
2.1	Equation of the y-axis	$x = k$ (k is a real number)
2.2	Equation of a line parallel to the y-axis	$y = 0$
2.3	Equation of the x-axis	$y = c$ (c is a real number)
2.4	Equation of a line parallel to the x-axis	$x = 0$

Why do this activity?

This activity engages learners in working out the gradients and equations of lines. They should notice that they get the same gradient whatever two points on the line they choose. The activity prompts learners to notice features such as when lines are parallel, where they cut the y-axis and when they have zero gradient or infinite gradient.

Learning objectives

In doing this activity students will have an opportunity to:

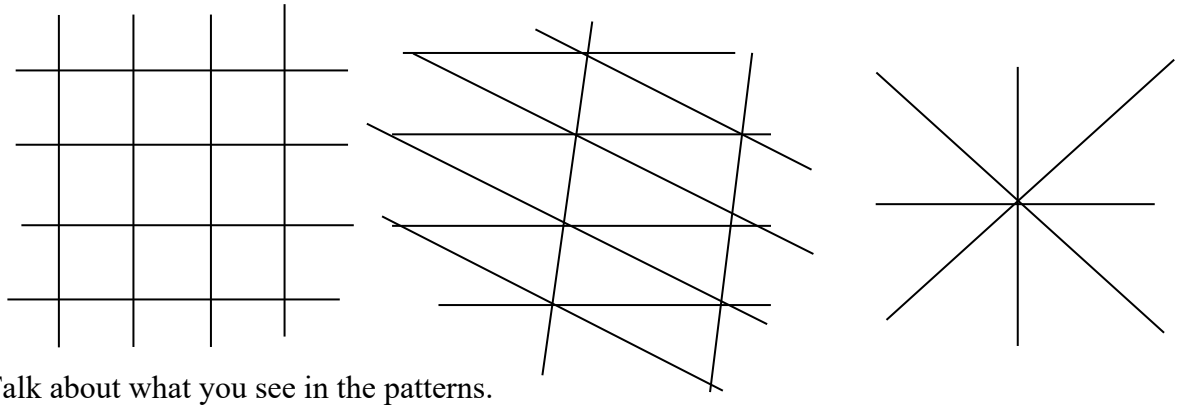
- use the Cartesian co-ordinates to derive and apply the gradient of a line through two given points;
- to investigate the equation of a line through two given points.

Generic competences

In doing this activity students will have an opportunity to **think flexibly**, be creative and apply knowledge and skills.

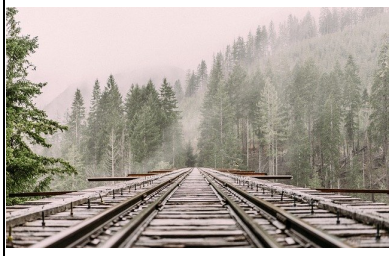
Suggestions for Home Learning

Young children Here are some patterns of straight lines. Can you draw some patterns?



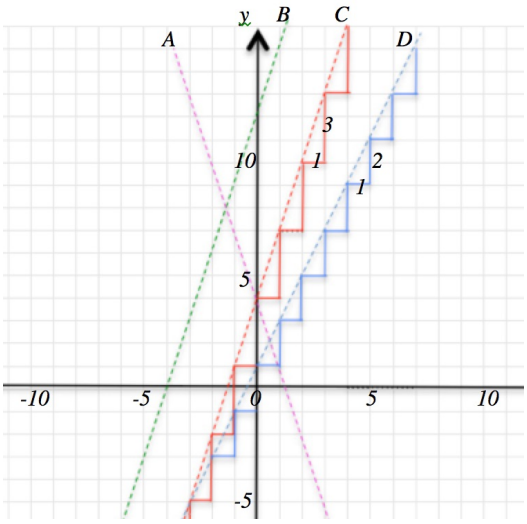
Talk about what you see in the patterns.

Upper Primary



Again the learners should draw patterns with lines and talk about the shapes that they see in the patterns, about parallel and perpendicular lines and about angles between lines. This is an opportunity to help the learners to develop fluency in mathematical language. Talk about straight lines and curved lines.

Lower Secondary



Talk about what you see in this picture.

The steps show the gradients of the lines.

Line C is steeper than line D.

The gradient = $\frac{\text{Rise}}{\text{Tread}} = \frac{3}{1} = 3$ for line C.

The gradient = $\frac{\text{Rise}}{\text{Tread}} = \frac{2}{1} = 2$ for line D.

What else can the learners see in the diagram?

They should draw a line with gradient 1 and another line with gradient 4.

See Steps https://aiminghigh.aimssec.ac.za/years-7-9-steps/

Years 9 and 10

You might print the worksheet on page 1 and ask learners to work individually or in pairs. You should emphasise that only 2 points are required for drawing a straight line.

While the learners are working observe how they are getting on. If they seem to be in difficulties then assist them by giving them the HELP slip, and later by asking Key Questions from page 5, but refrain from telling them what to do.

After the learners have completed the work then have a discussion when they learners explain their working and their solutions. At the end of the lesson you might wish to do the Diagnostic Quiz to find out how much the learners have understood about gradients.

Years 11, 12 and 13

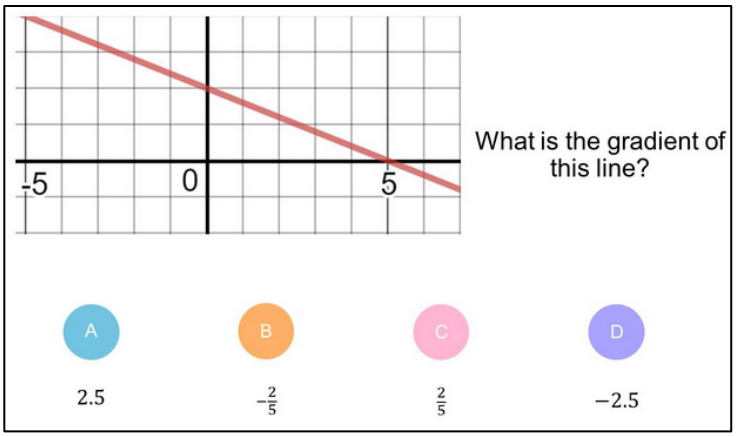
These young people should do the question discussed above for Years 9 and 10 and then the two questions in the NEXT section.

Key questions

- What do the changes in x and y between two points tell you about the slope or gradient of the line through the points?
- What happens in that equation when $x = 0$ and where is the point on the graph?
- What is the equation of the y-axis?
- What is the equation of the x-axis?
- If two lines have the same gradient what can you say about them?
- How do you know when two lines are parallel?

Diagnostic Assessment This should take about 5 minutes.

1. Write the question on the board and ask the class to
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



2. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank them for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class **again** to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. As the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

B. is the correct answer.

Common Misconceptions

A. Here the learner is confused between x and y and is dividing 5 by 2.

C. Learners may give this answer when they know it is rise/run but they get 2/5 because they work out (2-0)/(5-0) instead of (2-0)/(0-5) and they don't recognise the negative gradient.

D. Learners who give this answer recognise the negative gradient but (as in A) they divide change in x or 'run' by change in y or 'rise' instead of rise/run. https://diagnosticquestions.com

Follow up

See: Steps https://aiminghigh.aimssec.ac.za/years-7-9-steps/
Graphical Triangle https://aiminghigh.aimssec.ac.za/years-8-10-graphical-triangle/
Odd one out https://aiminghigh.aimssec.ac.za/years-7-9-odd-one-out/
Coordinate Patterns https://aiminghigh.aimssec.ac.za/years-10-12-coordinate-patterns/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6