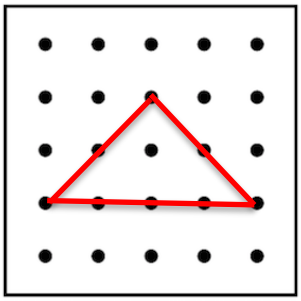
FIND THE RIGHT TRIANGLES

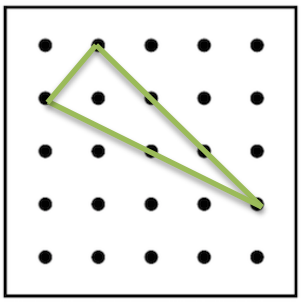


How many right-angled triangles of different shapes and sizes can you make by joining the dots on the 5 by 5 spotty grid?

Describe your triangles and say what geometrical properties they have.

HELP

Use the corner of a sheet of paper or cardboard to check that angles are right angles. This is particularly helpful when the sides of the triangles are not parallel to the edges of the grid.



The green triangle on this grid has an angle of 90° and the other two angles are about 75° and 15°.

How many more right-angled triangles can you find?

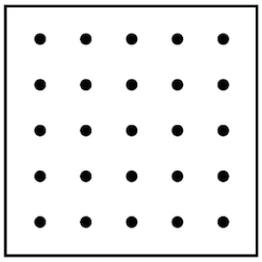
NEXT

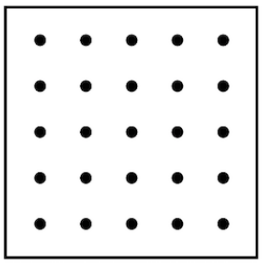
Can you find 16 right angled triangles of different shapes and sizes?

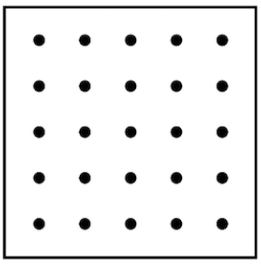
Find the areas of your triangles.

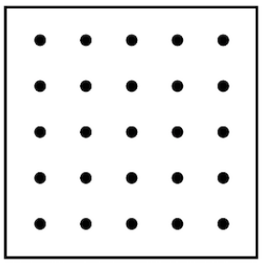
Can you find all the isosceles triangles of different sizes and shapes? There are 8 right angled isosceles triangles but there are also isosceles triangles that are not right angled.

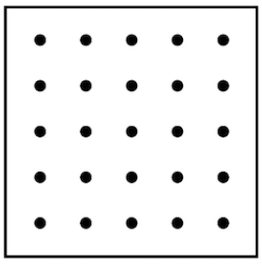
How many right-angled triangles of different shapes and sizes are there on a 5 by 5 grid?

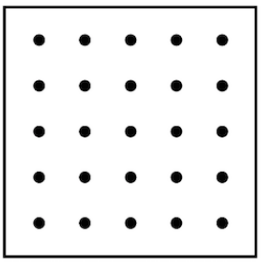


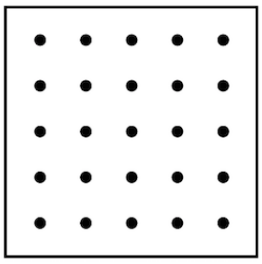


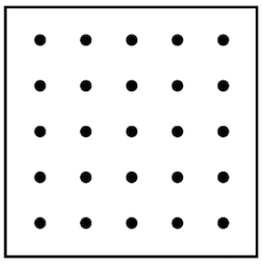


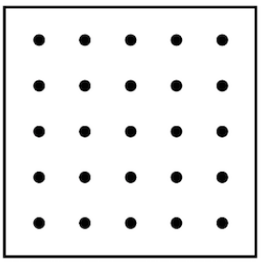


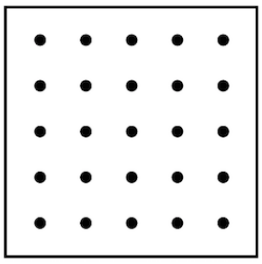


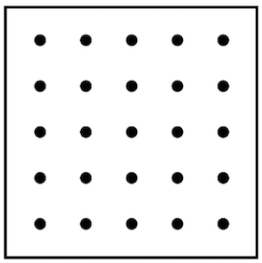


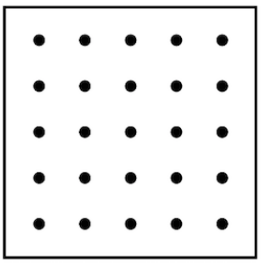


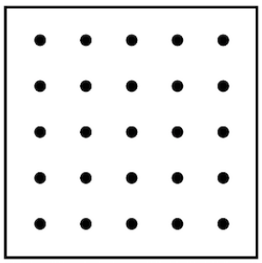


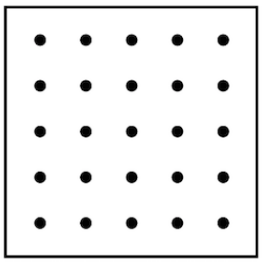


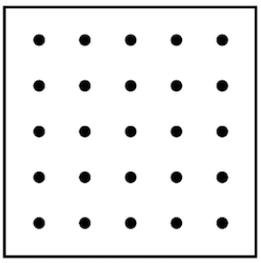


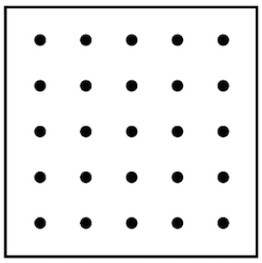










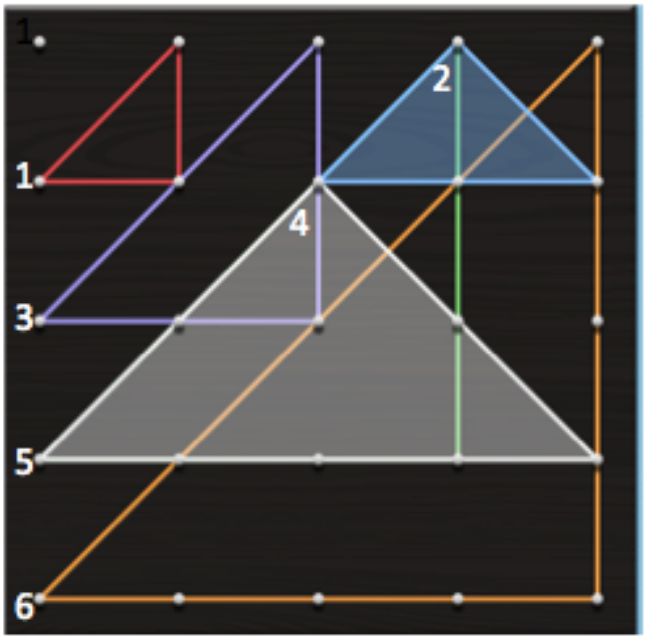


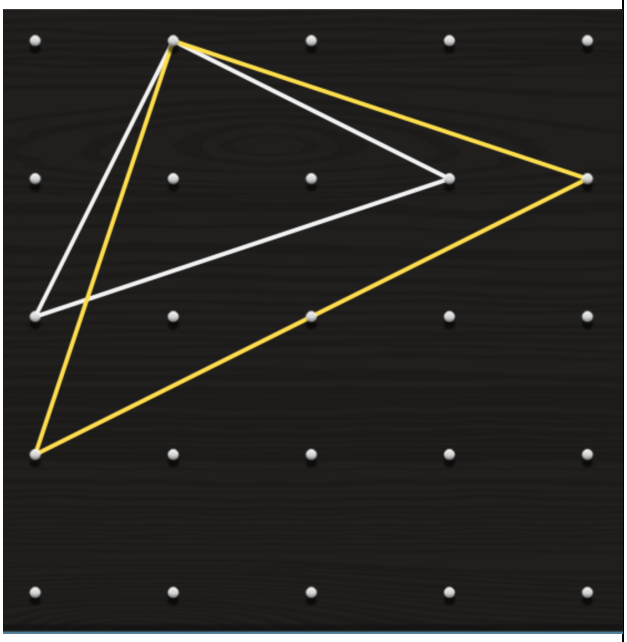
NOTES FOR TEACHERS

SOLUTION

This question does **NOT** ask "how many right-angled triangles of different sizes are there?" but "how many … can you make…" The activity is suitable for all ages. Any number of triangles is an acceptable answer.

If the teacher asks learners to find areas they can count squares and half squares. To find some of the areas may require a subtraction method. Older learners might use Pythagoras Theorem.





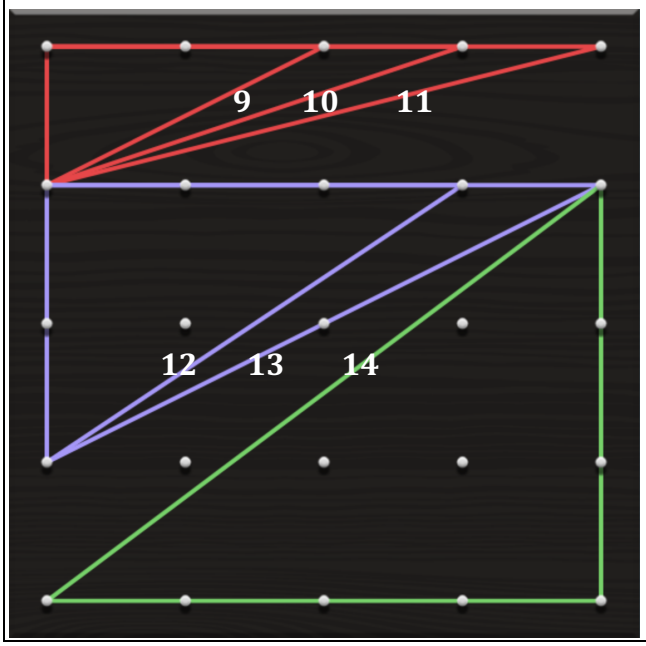
The diagrams above show isosceles right-angled triangles, that is triangles with two equal sides and angles of 45°.
All 8 triangles are similar to the red triangle and are enlargements by linear scale factors:

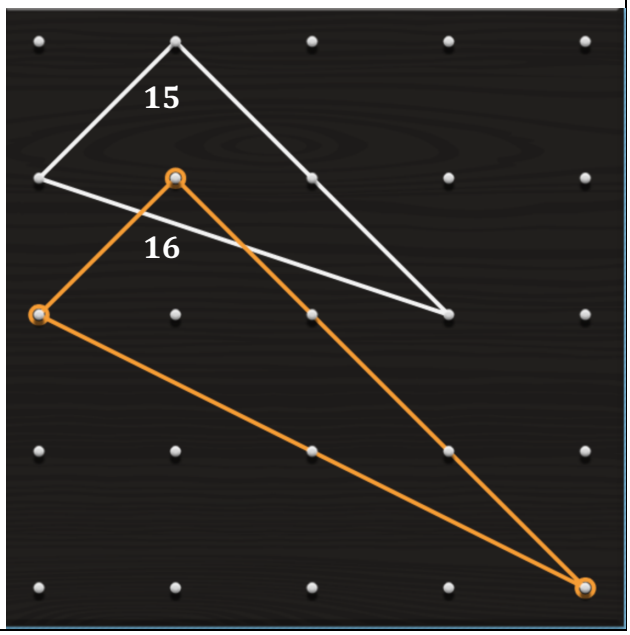
Triangle	1	2	3	4	5	6	7	8
Colour	Red	Blue	Lilac	Grey	Green	Orange	White	Yellow
Linear scale factor	1	√2	2	2√2	3	4	√5	√10
Edge lengths	1, 1, √2	√2, √2, 2	2, 2, 2√2	2√2,2√2,4	3, 3, 3√2	4, 4, 4√2	2√2,2√2,√10	3√2,3√2,√18
Area	½	1	2	4	4½	8	2½	5

All other right-angled isosceles triangles are **congruent** to one of these 8 triangles and the same size, just in some other position on the grid.

The right-angled triangles 9 to 16 on the next page are not isosceles. In each triangle the sides are different lengths and the angles are different.
The areas are given in the table below.

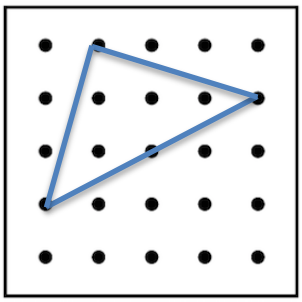
Triangle	9	10	11	12	13	14	15	16
Edge lengths	1, 2, √5	1, 3, 10	1, 4, √17	2, 3, √13	2, 4, √20	3, 4, 5	√2, 2√2, √10	√2, 3√2, √20
Area	1	1½	2	3	4	6	2	3





Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The area of this triangle is **not:**

A. $12 - \frac{3}{2} - \frac{3}{2} - 4$ sq units

B. $(\sqrt{10} \times \sqrt{10})/2$ sq units

C. $(\sqrt{10} \times \sqrt{5})$ sq units

D. $(\sqrt{20} \times \sqrt{5})/2$ sq units

The area is not given by C.

A. Boxing in the triangle with a 3 by 4 rectangle and subtracting the areas of the squares outside the given square give the area 5 square units.
B. The lengths of the edges are $\sqrt{10}, \sqrt{10}, \sqrt{20}$ by Pythagoras so it's area is half that of a square of area 10 sq units.
C. Not the area.
D. This calculation uses half base times height where the base is $\sqrt{20}$ and the height is $\sqrt{5}$.

Why do this activity?

This is a very open-ended question and suitable for all ages and attainment levels. Learners can use a geoboard or spotty paper to make triangles. All learners will have some success at drawing triangles. They can share their discoveries and, as a class, build up a collection of the different triangles that they have found. The teacher can guide and help them to develop the mathematical language needed to describe the geometrical properties that they should recognise at their level.

When two learners find the same triangle in different places on the grid the class can discuss how they are the same (same angles, same sides therefore congruent), how they are different, and how one can be transformed to the other (by translation, rotation and/or reflection).

Looking for right angles involves distances across and up between two points and builds familiarity with a grid system laying the foundation for work on coordinates, Pythagoras Theorem and the distance formula.

The teacher can ask the learners to find areas or lengths of edges if s/he wants to include this in the activity.

Learning objectives

Primary

In doing this activity students will have an opportunity to:

- draw 2-D shapes on grid paper;
- recognise and describe right angles and angles less than and greater than a right angle.

Lower secondary

- work out areas of triangles by different methods;
- recognize similar triangles and scale factors;
- recognize congruent triangles;
- investigate transformations that map triangles to congruent copies of themselves;
- identify and write clear definitions of triangles in terms of their edges and angles, distinguishing between: equilateral triangles; isosceles triangles and right-angled triangles;
- recognize and describe transformations between simple geometric figures on a co-ordinate plane;
- develop and use the theorem of Pythagoras.

Upper secondary

- recognize and describe angles formed by perpendicular lines;
- conditions for lines to be perpendicular;
- the distance between two points using coordinates.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- **persevere and work systematically** to investigate all possible cases.
- **work in a team** to produce a display of the work done by the class.

Suggestions for teaching

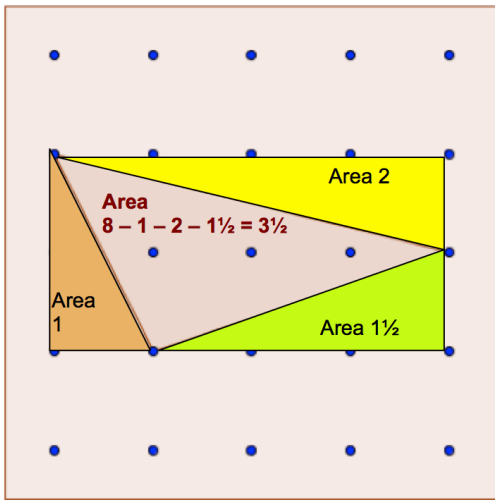
Choose which of the learning objectives you want to cover in the lesson. You might start with formative assessment where you draw some triangles on the board and ask learners to say what is the same and what is different about pairs of the triangles. Or if you want to work on area you might use the Diagnostic Quiz given on page 4.

Either use geoboards and make triangles with rubber bands, or give learners dotty grids as on page 2, so that they can draw triangles by joining the dots. It is best to use pencils so that they can erase mistakes.

Ask learners to make right-angled triangles of different shapes and sizes. When learners find the same triangle in different places on the grid ask "what is the same?" and "what is different?" and "how could you transform that triangle to that one?"

Make a big poster for the classroom wall using flip chart paper or print and enlarge page 2. Draw each new triangle as a learner finds one that is not already on the poster. The learners can be encouraged to go on looking for more triangles over a period of days or weeks until the class finds all 16 triangles.

The activity offers the opportunity to talk about similarity and congruence of triangles, to identify isosceles triangles and also to work on recognizing and describing transformations between two congruent triangles in different positions on the grid.



You might want the learners to work on finding areas. They do **not need** the formula for the area of a triangle. For example in this diagram the brown triangle is half of a 2 by 1 rectangle so it has area 2 square units, the yellow triangle is half of a 4 by 1 rectangle so it has area 2 square units, and the green triangle is half of a 3 by one rectangle so it has area 1½ square units.

To find the area of the triangle in the middle notice that it is enclosed by a 2 by 4 rectangle of area 8 square units so, to find the area of the inner triangle we need to subtract the areas of the other three triangles to give an area of 3½ square units. This is not a right angled triangle so the example shows that the method works generally and you do not need to find the lengths of the sides of the triangle.

You might choose to use this activity for learners to find lengths of edges of right-angled triangles using Pythagoras Theorem.

This is an excellent activity for formative assessment in Year/Grade 10 so that teachers can find out how well learners remember and understand the learning objectives from earlier years. The activity provides learners with practice in identifying and describing properties of triangles, recognising which lines are perpendicular, and finding distances and areas. Teachers can follow the same teaching approach as above for earlier grades and ask questions to that guide learners to review all these topics.

The activity can also lead to work on coordinates. In particular it is important to notice the distance across and up between two points which is important for finding gradients, distances and perpendicular lines.

Key questions

- How do you know that is a right-angled triangle?
- What is the same about those two triangles and what is different? Why?
- Looking at 2 congruent triangles on the same grid:
    - Show me which triangles are congruent. Why?
    - Could you slide (translate) that triangle to move it on top of the other one?
    - To move that triangle to go on top of the other one do you need to rotate it?
    - To move that triangle to go on top of the other one do you need to turn it over (flip or reflect) it?
- Looking at 2 similar triangles:
    - Show me which triangles are similar.
    - Show me the angles that are the same in both triangles.
    - How many times longer are the sides in that triangle compared to that one? (Linear scale factor).
    - Show me two triangles with one an enlargement of the other.
- Imagine a rectangular box around that triangle. How many squares are in the box? What is its area?
- Are any of the sides in that triangle the same length?
- Are any of the angles in that triangle the same?

Follow up

Pythagoras Jigsaw https://aiminghigh.aimssec.ac.za/grades-8-to-12-pythagoras-jigsaw/
Simsets https://aiminghigh.aimssec.ac.za/grades-8-to-10-simsets/
How Many Triangles Puzzle https://aiminghigh.aimssec.ac.za/count-the-triangles-puzzle/
Counting Triangles https://aiminghigh.aimssec.ac.za/years-7-10-counting-triangles/
Kite in a Square https://aiminghigh.aimssec.ac.za/years-9-to-11-kite-in-a-square/
Reflecting Squarely https://aiminghigh.aimssec.ac.za/grades-9-to-12-reflecting-squarely/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.
For resources for teaching A level mathematics see https://nrich.maths.org/12339

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6