COLA CAN

An aluminium can contains 330 ml of cola.

If the can's diameter is 6 cm what is the can's height?

If instead the can's height was 10 cm what would the can's diameter have to be?

Which of these two cans uses the least aluminium?

If you could choose any diameter which dimensions for the can would use the least amount of aluminium to hold 330 ml of cola?

Help

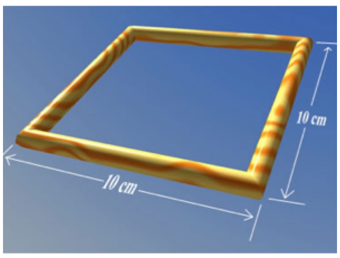
Here is some guidance for students who need extra help.

You need to know that a millilitre is one cubic centimetre then work through the following steps:

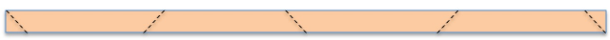
1. Check that you can calculate the volume of a cylinder.
2. If the diameter is 6 cm, you need to know how to calculate the base area of the can.
3. If you know the base area you can find a height which will give you the volume (330 ml in this example)
4. Now start with a height (10 cm) and work your way back to a base area for a given volume, then find a radius for that base area, and then find a diameter for the can.
5. Think what makes up the surface area, it has to be the aluminium. So you have to make the surface area as small as possible.
6. The top and base of the can are circles but how do you calculate the curved surface area?

Extension

See https://aiminghigh.aimssec.ac.za/years-10-12-woodwork/

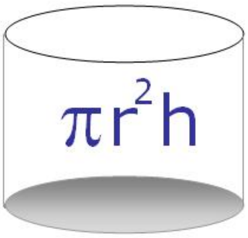


This stand is made from cylindrical wooden dowel of diameter 1 cm joined at the corners with 45 degree mitres.
The external dimensions of the stand are 10 cm by 10 cm.
What are the volume and surface area of the stand?
The sketch below shows how I marked out the dowel before cutting it. What length of dowel do I need to make this stand?



Find a formula linking the volume, radius and external edge measurement of the stand.

NOTES FOR TEACHERS



SOLUTION

The volume of a cylinder is found from the area of the circular cross-section multiplied by the height of the can and here the volume has to be 330 ml.

So when the can's diameter is 6 cm (r=3) it's height (h) is 11.67 cm.
And when it's height is 10 cm, the can's diameter has to be 6.48 cm.

We notice that if the volume is fixed then whether we know the can's diameter or it's height, the other of these two will be easy enough to calculate using the formula:

$$h = \frac{330}{\pi r^2} \quad \text{OR} \quad r = \sqrt{\frac{330}{\pi h}}$$

So whichever we already have, either r or h, we can find the other and then use both in the surface area formula for two circles, top and bottom, plus a rectangle (can circumference by can height - like peeling the label off a tin).

$$2\pi r^2 + 2\pi rh$$

We are now going to work systematically to find the diameter and height of the can (volume 330 ml) that has the least surface area.

Start with a radius of 3 cm and find the height that makes a volume of 330 ml, then use both those r and h values to calculate the surface area for the can.

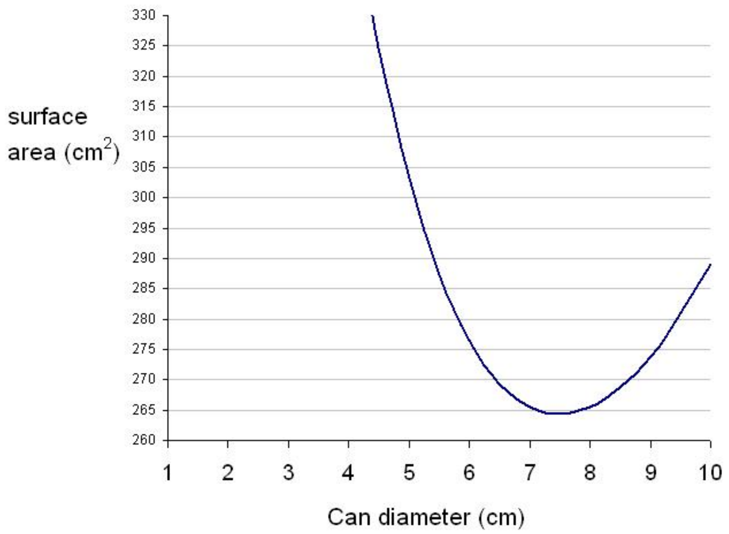
Diameter (cm)	Radius (cm)	Height (cm)	Surface area (cm^2)
6	3	11.67	276.55
7	3.5	8.57	265.54
7.3	3.65	7.88	264.53
7.4	3.7	7.67	264.40
7.5	3.75	7.47	264.36
7.6	3.8	7.27	264.41
8	4	6.57	265.53
9	4.5	5.19	273.90
10	5	4.20	289.08

Then increase the radius and repeat the calculations.

We could continue to increase the radius and make up a table of the surface area results each time.

Alternatively we could use the same method but start with a value for height instead, and increase that.

Once we see how the surface area changes with radius (or height) in a general way we can refine our choice of an r or h value until, by trial and improvement, we get as near as we please to the lowest value for the surface area.



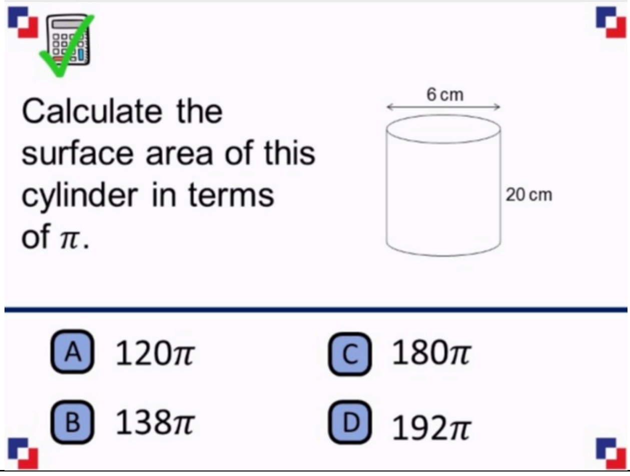
Here's a graph showing an interesting and non-symmetric curve for the changing surface area.

So the can with diameter about 7.5 cm has the minimum surface area.

Here's a link to an Excel file that quickly does all the calculations: Cola Can

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is B
Possible misconceptions:

A. Students giving this answer probably multiplied the numbers given $6 \times 20 \times \pi$

C. Probably a guess

D. One student said he estimated $3 \times 20 \times \pi$ to get 192.

https://diagnosticquestions.com

Why do this activity?

The activity gives an opportunity for students to work on a problem that has a 'real life' application. Students are led step by step to see that, if the capacity of the can remains the same, then the independent variables, the base radius r and the height h, are connected by a formula. The first part of the question, which involves starting with either of these variables, calculating the other and then calculating the surface area, is routine. This opens the way for higher order thinking about continuously varying the dimensions and minimizing the surface area. This is a rich activity offering scope for working with a table of values and drawing a graph, even possibly setting up a spreadsheet to do the calculation.

Learning objectives

In doing this activity students will have an opportunity to:

- practise calculations of volume and surface area of a cylinder.
- solve a problem involving a function of two variables with constraints, and looking for a minimum value.

Generic competences

In doing this activity students will have an opportunity to:

1. think mathematically
2. be creative and innovative - to apply knowledge and skills
3. develop the skill of interpreting and creating visual images to represent concepts and situations
4. interpret information
5. solve and interpret problems in a variety of situations

Suggestions for teaching

Make sure that the students know that a millilitre is one cubic centimetre. As preliminary formative assessment use the diagnostic question to check that students can work out the surface area of a cylinder and then check that they can calculate the volume of the same cylinder. Showboards are useful for this.

Then write the question on the board or give out a worksheet copied from the top half of page 1. If learners work in pairs or small groups they can share the work of making a table of values for the surface area for different can diameters.

You might guide learners to draw a graph or to compile a spreadsheet.

Key questions

- If you know the volume of the cylinder is 330 what formula can you find that connects r and h?
- Can you make r the subject of that formula?
- Can you make h the subject of that formula?
- If you know the volume is 330 can you start with the diameter and work out r and h and the surface area?
- Can you make a table of those values and draw a graph to show how the surface area varies as you change the diameter?

Follow up

Woodwork: https://aiminghigh.aimssec.ac.za/years-10-12-woodwork/
For another activity involving cylinders see the problem Funnel http://nrich.maths.org/5890

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6