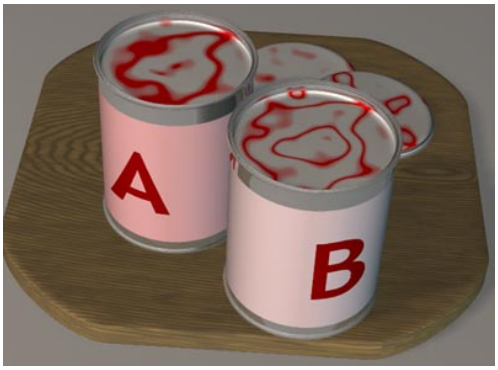
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

MIXING PAINTS



Paint A is made up from red and white paint in the ratio **1:3** and paint B is made up from red and white paint in the ratio **1:7** . The cans are the same size.

You can mix the paints to produce different shades of pink. Explain how to find the ratio of red paint to white paint if you mix one can of A with one can of B.

What is the ratio of red to white if 1 can of A is mixed with 2 cans of B? What about mixing one can of A with 6 cans of B?

What is the least number of cans of each type needed to produce pink paint containing red and white in the ratio **1:4** ?

Help

Imagine a chocolate bar that has 8 pieces.

What does it mean to share it in the ratio 1 : 3? How many pieces in each share? What about sharing in the ratio 1 :7?

If you shared two bars, one bar in the ratio 1 : 3 and the other in the ratio 1 : 7 how many pieces would each person get?

Extension

Is it always possible to combine two paints made up in the ratios **1 : *x*** and **1 : *y*** and turn them into paint made up in the ratio **1 : *z*** (where $x < z < y$)? To investigate this further think about another painter and decorator who buys pink paint from two different manufacturers:

- Paint C is made up from red and white paint in the ratio **1:4**
- Paint D is made up from red and white paint in the ratio **1:9**

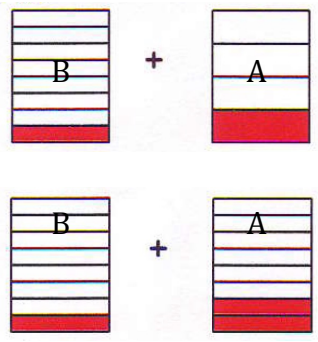
What is the least number he would need of each type in order to produce pink paint containing red and white in the following ratios:

- 1 : 5
- 1 : 6
- 1 : 7
- 1 : 8

For interactive versions of this question and a chance to mix your own paints see:
http://nrich.maths.org/4793 and http://nrich.maths.org/4794

NOTES FOR TEACHERS

SOLUTION



Paint A has red to white in the ratio 1 : 3 that is ¼ red and ¾ white.
Paint B has red to white in the ratio 1 : 7 that is ⅛ red and ⅞ white.
The parts compared in any ratio must be the same size so we need to work in ⅛ ths.

Mixing one can of each type of paint would give the ratio ⅜ red and 13/8 white so the ratio in this mixture is 3 parts red to 13 parts white or 3 : 13 each part being one eighth of a can.

A common mistake here is to combine the ratios 1 : 3 and 1 : 7 and to say that mixing one can of each type of paint would give the ratio 2 : 10 or 1 : 5. This is wrong.

If 1 can of A is mixed with 2 cans of B we get 4 parts red and 20 parts white giving the ratio 4 : 20 that is 1 : 5 .

If 1 can of A is mixed with 6 cans of B we get 8 parts red and 48 parts white giving the ratio 8 : 48 that is 1 : 6 .

If 3 cans of A is mixed with 2 cans of B we get 8 parts red and 32 parts white giving the ratio 8 : 32 that is 1 : 4 .

We can use algebra to work this out. Suppose x cans of A and y cans of B are used then this gives 2x +y parts red and 6x + 7y parts white. Looking for the ratio 1 : 4 we need $4(2x + y) = 6x + 7y$ that is $2x = 3y$.

The smallest solution to this is x = 3 and y = 2, that is x cans of paint A and y cans of paint B.

Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The ratio of red to yellow flowers is 5:3, the ratio of blue to red flowers is 1:10. What is the ratio of blue to yellow flowers?

The correct answer is B.
If we changed the yellow to red ratio to 10 : 6 (still the same), we can combine the two ratios, which makes blue : red : yellow = 1 : 10 : 6 so the ratio of blue to yellow is 1 : 6.

Possible misconceptions:

A. The learner has tried to combine the 3 ratios but done this incorrectly.

C. The learner has combined the 3 ratios but not answered the question that asked for the ration of blue to yellow.

D. The learner has used the numbers 1 and 3 from the given information but has not understood the concept of ratios.

https://diagnosticquestions.com

Why do this activity?

This activity gives learners a real context in which to interpret the meaning of statements involving ratios and it gives practice in working with ratios. Thinking about how to answer this question will help learners to overcome the misconception that combining one can of each type gives a ratio of 2 : 10 . The diagrams help in clearing up this misconception and teachers can scaffold the learning to help learners to think along the right lines. In addition this activity can be used to give learners practice in reading and interpreting questions for themselves.

Learning objectives

In doing this activity students will have an opportunity to review, appreciate and find meaning in the concept of ratios.

Generic competences

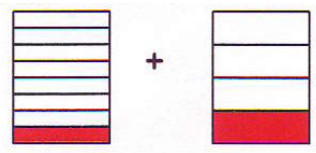
We need to prepare children for a job market where existing knowledge and skills have limited value unless they can be applied in novel ways to produce new knowledge that solves today's complex problems to improve the quality of life for all.

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **apply** knowledge and skills;
- **visualise** - develop the skill of interpreting and creating visual images to represent concepts and situations;
- **interpret and solve problems** in a real life context;
- **work and learn independently** and prepare for lifelong learning;
- **work in a team**:
  - co-operate - to collaborate/work with a partner or group
  - have empathy with others, listen to different points of view
  - develop leadership qualities
- **communicate** in writing, speaking and listening:
  - exchange ideas, criticise, and present information and ideas to others
  - analyze, reason and record ideas effectively.

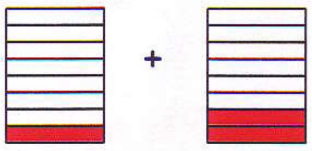
Suggestions for teaching

Starting lessons with a diagnostic quiz gives all learners the opportunity to review what they have previously learned about topic for that lesson. Teachers can boost the confidence of the learners who give one of the wrong answers by reminding them that research has proved that making mistakes and learning from our mistakes makes our brains develop by strengthening the neural pathways so we can be more successful in future.



You could use the **1 – 2 – 4 - more** teaching strategy. Give learners a few minutes to read the question individually and to try to start answering the question. Draw this diagram on the board and tell learners to try to use it to help them to answer the first part of the question. After a few minutes ask learners to discuss this question and try to do it with a partner. Then ask the learners to draw the diagram in their notebooks and to label it. Then ask what the two parts of the diagram represent, and what fractions are shaded.

Then have a class discussion about the first part of the question. You may find that some learners suggest that the answer is 1 : 5 and other learners have found the answer 3 : 13. You can ask learners to explain how they found their answers and perhaps ask the class to vote for one answer or the other. Ask "are the parts the same size?" and ask what they need to do to make the parts from can A and the parts from can B the same size. Then draw and label the second diagram.



Then give the class time to work in pairs to complete the question and then to compare and discuss their answers in groups of four. This enables the learners to check their answers and to try to reconcile answers if they disagree thus correcting their own misconceptions. Having given time for all learners to think for themselves the **1 – 2 – 4 - more** strategy enables the learners to explain the ideas to each other and to help other learners who are having difficulties.

Finally work with the whole class to go over the question so that the learners can check their working. Some learners will get the ratios right but others may have made the same mistake as discussed above. This enables the teacher to help all learners to understand that when combining ratios they must be working with parts of the same size and to clear up a common misconception. This class discussion, after the learners have worked on the activity, enables the teacher to summarize what has been learned based on the learners' own experience, and to reinforce a clear understanding of ratios.

For the last part of the question most learners will use a trial and error method, and some may not find the answer. Some learners may have worked out the last part using algebra. According to the class and the time available the teacher can suggest the algebraic method and ask learners to explain it to the class if anyone has done it that way.

Key questions

- What does the question tell you?
- What fraction of the paint in the can is red?
- What fraction of the paint in the can is white?
- Mixing the cans of paint how many parts of red paint do you get and how many parts of white paint?
- Are the parts the same size? How do you know?

Follow-up ideas

An easier question - Pizza https://aiminghigh.aimssec.ac.za/grades-8-and-9-pizza/
More challenging and involving similar triangles – Wedge on Wedge
https://aiminghigh.aimssec.ac.za/grade-10-or-11-wedge-on-wedge/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6