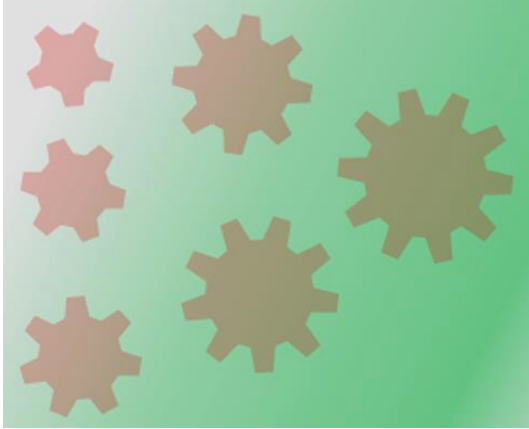


TURNING COGWHEELS



You can experiment with your own cogs made with scrap card using this diagram.

Pick two cogs. Place them close to each other so that the teeth fit into the gaps between the teeth of the other cog and as you rotate one it makes the other one rotate. What do you notice about the direction in which the cogs turn each time?

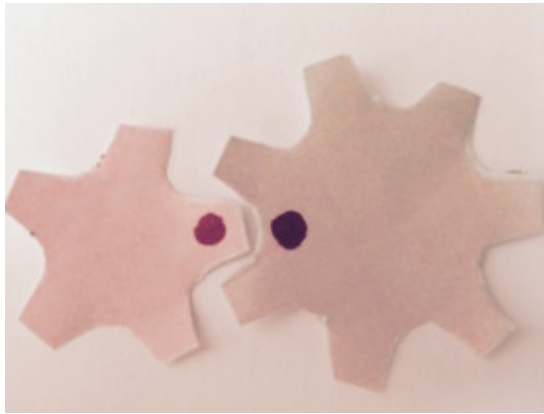
Mark a spot on each cog next to each other as shown. What can you say about the way that the teeth of one cog fit into the gaps of the second cog?

What happens to the spots as the cogs rotate? Do the spots meet again? If so when, and why?

Will the spot on the 5-cog go into every gap on the other cog?

Now pick two cogs with 5 teeth and do the same again. What do you notice? Explain what happens.

Now pick a different pair of cogs and do the same thing again. Does the same thing happen every time whichever two cogs you pick?



Help

Make your own cogwheels from scrap card or cut out some cogwheels from the template on page 2. Follow the instructions and record what happens.

Working with a partner or in a group and help each other. Ensuring that everyone understands before the group moves on.

Extension

Make up some of your own questions to investigate or work on questions posed by another group.

NOTES FOR TEACHERS

RESOURCES Cut out a set of cogwheels for each group of 4 students using the sheet on page 5 and give them out so that each person in a group has two or three cogwheels.

SOLUTION

One cogwheel rotates clockwise and the other one anti-clockwise.

When the 5-cog has turned around 7 times and the 7-cog has turned around 5 times the spots meet again. This is because the tooth on the 5-cog meets every 5th gap on the 7-cog and the two spots coincide at the 35th gap because 35 is the lowest common multiple of 3 and 5. The spot on the 5-cog will go into every gap on the other cog.

With two 5-cogs the two spots meet after each revolution and the spot on one cog will only go into one gap on the other cog.

With a 3-cog and a 6-cog the spots will meet when the 3-cog has turned around twice and the 6-cog has turned once. This is because the tooth on the 3-cog meets every 3rd gap on the 6-cog. If the spots are numbered 1, 2, 3, 4, 5, 6 the spot on the 3-cog will only go into two gaps 1 and 4 (or 2 and 5, or 3 and 6).

For any 2 cogs, the spot on one will go into every gap in the other cog if the number of teeth have no common factors.

Extra information for teachers:

Suppose a p-cog and a q-cog turn together where $p < q$ where p and q have no common factors and p is not a factor of q. Then the spots will meet after q revolutions of the p-cog and p revolutions of the q-cog.

If p and q have a common factor, say $p = rt$ and $q = st$ then $ps = rq$ and so the spots meet after s turns of the p-cog and r turns of the q-cog.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Which number in the list is a common multiple of 8 and 12?

- A. 16
- B. 4
- C. 36
- D. 48

The correct answer is D

Possible misconceptions:

- A. This answer shows poor understanding as 16 is not a multiple of 12.
- B. Students giving this answer may think the question is about factors and don't understand either concept.
- C. This answer shows poor understanding as 36 is not a multiple of 8.

<https://diagnosticquestions.com>

Why do this activity?

This activity makes learners think about factors and multiples and, in particular, common factors, but it is not necessary for them to have met this term prior to having a go at the task. It offers opportunities for learners to ask their own questions, find examples, make conjectures and begin to generalise.

The activity lends itself to collaborative working, both for learners who are inexperienced at working in a group and those who are used to working in this way. By working together on this problem, the task is shared and therefore becomes more manageable than if working alone.

Learning objectives

In doing this activity students will have an opportunity to work make sense of multiples and common multiples through investigation of how cogwheels work.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically and flexibly**, reason logically and give explanations and proofs;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts;
- **persevere and work systematically** to investigate many typical cases;
- **investigate and pose and solve problems**;
- **work in a team**:
 - collaborate and work with a partner or group
 - have empathy with others, listen to different points of view
 - develop leadership qualities;
- **communicate** in writing, speaking and listening according to the audience:
 - exchange ideas, criticise, and present information and ideas to others
 - analyze, reason and record ideas effectively.

Suggestions for Teaching

Tell the class that they are going to investigate cogs which are very important for gears in machinery such as clocks, bicycles, cars and engines.



Cut out a set of cogwheels for each group using the sheet on page 5 and give them out so each person in a group has two or three cogwheels.

This is an ideal activity for learners to tackle in groups of four. You may like to allocate roles to each learner (see below). Give each group a copy of these questions which describe the task. The idea is for them to read it together to find out what to do.

Explain that each group will be expected to report back at the end of the session, showing the patterns they noticed, at least one conjecture they have made, and at least one question. Exploring the full potential of this task is likely to take more than one lesson, allowing time in each lesson for learners to feed back ideas and share their thoughts and questions.

Counting Cogs *Read this aloud in your group first!*

Main Question: Which pairs of cogs let the tooth with a spot go into every gap on the other cog? Which pairs do not let this happen? Why?

Can you explain how to find which pairs of cogs will work, and why?

Each member of your group will get two or three cogs to investigate.

Before you begin, make a spot on one tooth on each cog.

1) Your task is to investigate the cogs you have. Choose one pair of cogs at a time and find out if the spotted tooth on one cog goes into every gap on the other cog.

2) When you have decided on that, record your work and swap one cog with another group member.

3) Investigate the new cog pair and repeat until all combinations of cogs have been investigated in your group.

4) Discuss any patterns you see with your group.

What questions do you have?

What conjectures can you make about why certain pairs work and others don't?

5) Record what you notice, any conjectures you have and any questions you would like to ask ready to report back to the whole class.

Ask each group to record their reasoning, conjectures, explanations and any generalisations on a large sheet of paper (for example flipchart paper) in preparation for reporting back.

There are many ways that groups can report back. Here are just a few suggestions:

- Two learners from each group move to join an adjacent group. The two "hosts" explain their findings to the two "visitors". The "visitors" act as critical friends, requiring clear mathematical explanations and justifications. The "visitors" then comment on anything they did differently in their own group.
- Every group is given a couple of minutes to report back to the whole class. Learners can seek clarification and ask questions. After each presentation, the other learners are invited to offer positive feedback. Finally, learners can suggest how the group could have improved their work on the task.
- Everyone's posters are put on display at the front of the room, but only a couple of groups are selected to report back to the whole class. Feedback and suggestions can be given in the same way as above. Additionally, learners from the groups which don't present can be invited to share at the end, to tell the class about anything they did differently.

Key questions

- Which cogs have you found that work so far?
- Which pairs didn't work? Can you explain why?
- How could you predict whether a pair will work before you try them?
- What questions would you like to ask?

Follow up

Simple practical investigations involving common multiples:

<https://aiminghigh.aimssec.ac.za/years-3-6-making-sticks/>

<https://aiminghigh.aimssec.ac.za/years-3-7-strip-patterns/>

<https://aiminghigh.aimssec.ac.za/years-5-9-path-to-the-stars/>

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.

For resources for teaching A level mathematics see <https://nrich.maths.org/12339>

Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6

