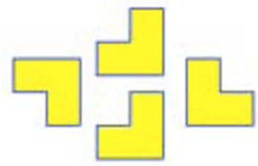
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

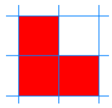
TRISQUARES

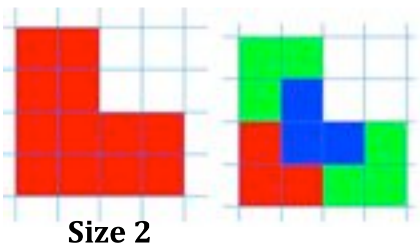


These 'trisquares' (also called tri-ominoes) are made up of three squares and each has an area of 3 square units.
Can you fit them together to make an enlargement of the shape? What is its area?
Can you fit trisquares together to make enlargements of scale factors 3, 4 and 5? What are their areas?
Is it possible to make enlargements of all sizes by fitting trisquares together?
Squared paper would be useful for working on this activity

Solution

A trisquare (or triomino) is a shape made from three squares (size 1).





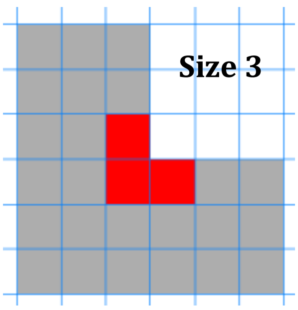
The diagram shows how 4 trisquares fit together to make a trisquare ENLARGED by a LINEAR SCALE FACTOR 2.
The shapes are SIMILAR because, when we compare the 2 shapes, all the angles are the same and each edge in the bigger shape is twice the length of the corresponding edge in the smaller shape.
The AREA SCALE FACTOR is 4. The areas are 3 squares and 12 squares. It might help to refer to them as 3-square and 12-square pieces.

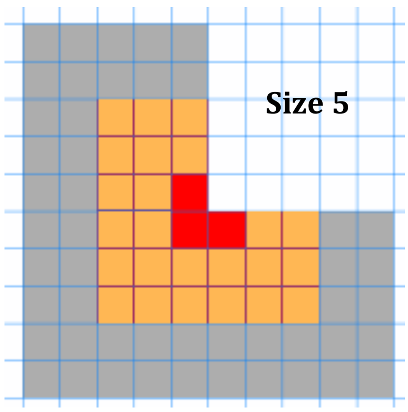
This is to help you to make a similar shape with area scale factor 16 using the tiling of 4 trisquares above (4 of the 12-square pieces). You might find it easier to do this next rather than to do the shape with linear scale factor 3, area scale factor $3^2=9$.

Remember:

Basic shape	Area 3 squares	
Linear scale factor 2	Area 12 squares	Area scale factor $2^2 = 4$
Linear scale factor 3	Area 27 squares	Area scale factor $3^2 = 9$
Linear scale factor 4	Area 48 squares	Area scale factor $4^2 = 16$
Linear scale factor 5	Area 75 squares	Area scale factor $5^2 = 25$ and so on

Can you see how to use 4 of the 12-square pieces outlined in the diagram above to make a similar shape (an enlargement) of area scale factor 16?
In this way the trisquares fit together to make enlargements of linear scale factors: 2, 4, 8, 16, 32 etc., that is corresponding to area scale factors: 4, $4^2=16$, $4^3=64$, $4^4= 256$, $4^5=1024$ and so on.





What about odd sized trisquares, for example linear scale factor 3 and area scale factor 9?
To make the enlargement by linear scale factor 3 shown, you use one 3-square piece, one 12-square piece and two 2x3 rectangular blocks (each made up of two 3-square pieces). Try it for yourself.

To make the enlargement by linear scale factor 5 you simply add another 12-square piece in the bottom left and for the rest you can use the 2x3 rectangular blocks. Try it and see.

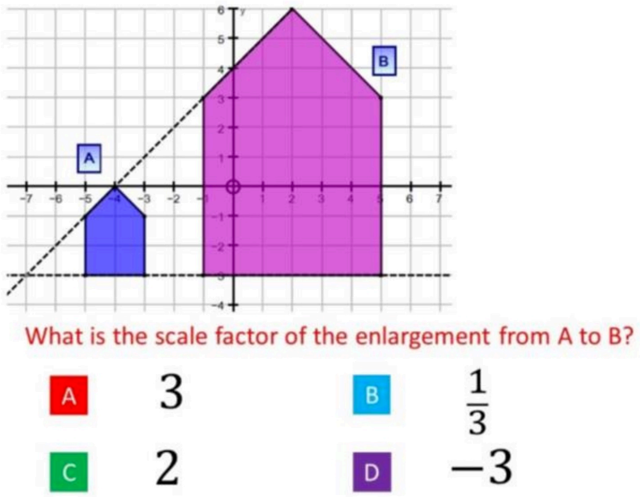
It's the same with the enlargement by linear scale factor 7, add a 12-square piece in the bottom left of the 75-square, size 5 piece and then use 2x3 blocks to tile the rest. This works with all odd numbers.

There is a little more work to do to show that you can also make enlargements of linear scale factors 6, 10, 12 etc (that is even but not a power of 4) so that enlargements of all linear scale factors can be made just by fitting together the basic 3-square.

Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



A. is the correct answer.

Common Misconceptions

B. These learners have given the scale factor for the enlargement from B to A.

C. Could just be a guess.

D. Does not understand the significance of a negative scale factor.

https://diagnosticquestions.com

Notes for teachers

Why do this activity?

This is a good 'low threshold high ceiling' activity. Some learners may just do some of the simpler tilings and, if they struggle, the outlines can be given to them on squared paper. The high flyers can be asked about extending their methods to discover if they can tile trisquares of all sizes.

This begins with very simple ideas about **similar shapes**, **enlargement** and **area.** It gives practice in using the ideas and language of **scale factors**. It can lead learners to appreciate how proofs can be built up in stages, through breaking an idea down into special cases. It also introduces the intriguing mathematical notion of reptiles – that is shapes that can be tiled to make enlargements of themselves.

Intended learning outcomes

Development of understanding of scale factors
Development of problem solving and visualisation skills.

Suggestions for teaching

Remember don't give learners the solution but be ready to give them some ideas of how they might start and then make progress. You want them to try and get to the solutions themselves with hints from you when they run out of ideas.

Start by introducing the smallest trisquare (L-triomino) and challenge learners to draw enlargements of linear scale factors 2, 3 and 4 on squared paper. Then challenge them to fit the basic 3-square piece into the outlines they have drawn like fitting together pieces of a jigsaw. . They can do this by pencilling in the outlines but if they find this difficult they could cut out some of the smallest 3-square pieces and try fitting them together in different ways.

They may work in quite a haphazard way to start off with, and may not even find ways of tiling them all. There is the opportunity to discuss the number of tiles needed, and to make links with work on enlargements and similar shapes.

Once everyone has had a go at tiling the first few trisquares (L-triominoes), and successful attempts are collected on the board for all to see, follow up with: **"I wonder whether all sizes of trisquare can be tiled"**

Suggest the need for a systematic approach, gradually building up knowledge of how different sizes of trisquare can be tiled.

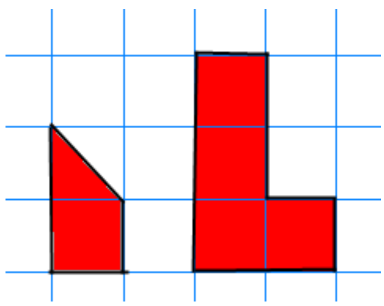
Ask learners to think about how any size trisquare can be enlarged by a linear scale factor 2. Can they develop their ideas further and suggest how they would convince someone that enlargements by 2^n for all values of n can be tiled? Bring the class together to share their insights.

Next, introduce the enlargements with odd scale factors. Share the diagrams in the solutions above with the class.

The class could make posters to illustrate (explain) how all trisquares can be tiled. One poster could show how the odd sized trisquares are made in sequence one leading to the next, and another poster could show how the 2^n sized trisquares are made by taking 4 of each one to make the next one. The posters can be made in groups or by the whole class to summarize the ideas and complete the lesson.

Key Questions

How can I use my knowledge of tiling a size 2 L-triomino to tile a size 4?
How can I use my knowledge of tiling a size 3 L-triomino to tile a size 5?
How can I use my knowledge of tiling odd and $2n$ sized L-triominoes to tile ANY size L-triomino?



Possible Extension

Investigate tilings with these other reptiles:

Come up with similar proofs that all sizes can be tiled.

Possible Support

A nice way of showing how the $2n$ sized trisquares (edges 2n, 2n, n, n ,n and n), can be built up is for each learner to create a size 2 trisquare, and then to stick four of these together in their group of four to make a size 4, and then for four groups to get together to make a size 8...

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6