

*What do you see in this diagram?*

*Look at the triangle with angles marked 1, 2 and 3. Call this T.*

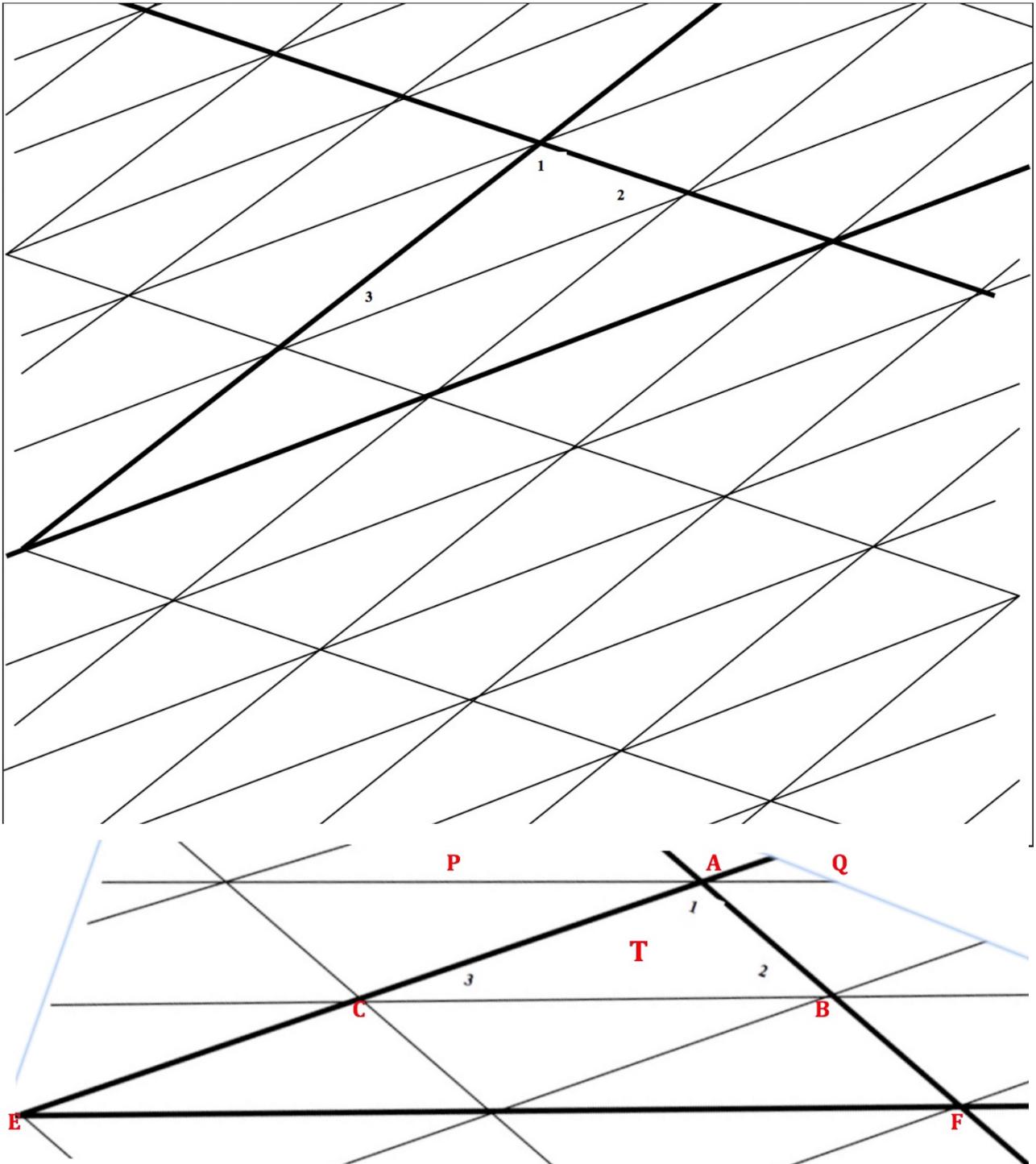
*In the enlarged part of the diagram at the bottom of the page label angles equal to these angles.*

*Why are they equal? What geometrical facts can you see illustrated in this diagram?*

*Look at the larger heavily outlined triangle.*

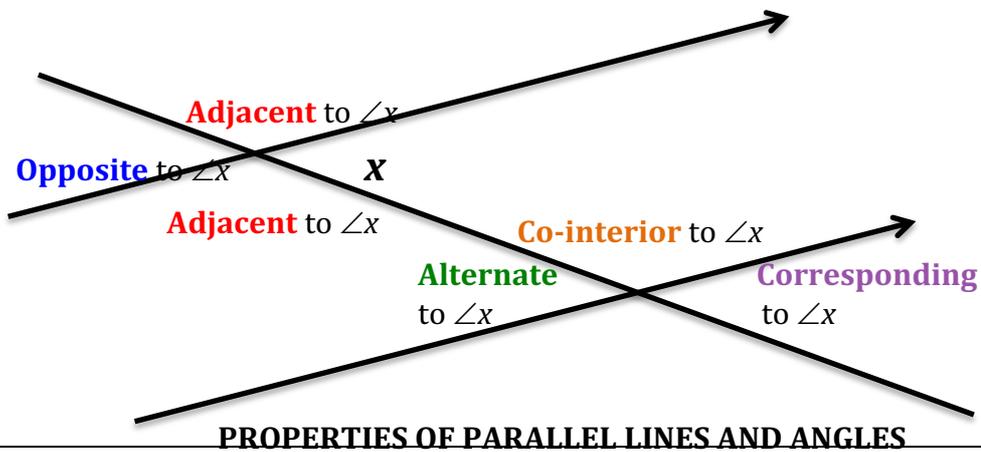
*How many copies of triangle T can you see in this triangle?*

*What else do you notice about this triangle?*



## HELP

It helps to work with a partner if you can. You might use a cardboard or plastic triangle cut from scrap material as a template, and draw around it to make your own tessellation like the one shown. You can refer to the diagram below to help you to know what words to use.

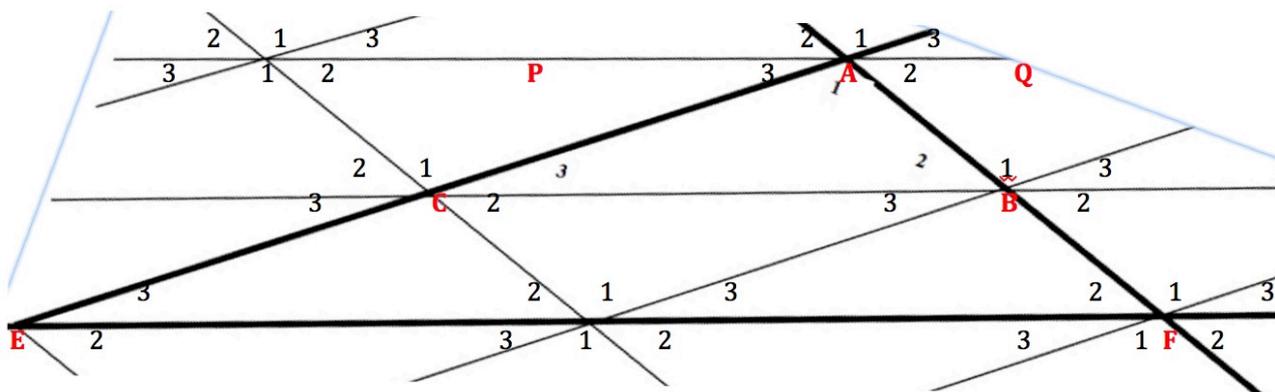


## NEXT

Draw your own tessellations using ruler, protractor and compasses.

## NOTES FOR TEACHERS

### SOLUTION



All the angles marked 1 in the diagram are equal to  $\angle CAB$  as vertically opposite angles are equal and corresponding angles between parallel lines are equal.

Similarly all the angles marked 2 are equal and all the angles marked 3 are equal for the same reasons. This labelling can be continued indefinitely in all directions across the whole tessellation.

The diagram illustrates the theorem that the angles of a triangle add up to  $180^\circ$ . The line PAQ is parallel to CB so the alternate angles  $\angle PAC = \angle ACB$  (angle 3) and  $\angle QAB = \angle ABC$  (angle 2). As the angles on a straight line add up to  $180^\circ$  so

$$\angle PAC + \angle CAB + \angle BAQ = \angle 3 + \angle 1 + \angle 2 = \angle ACB + \angle CAB + \angle ABC = 180^\circ$$

The  $\triangle ABC$  (also labelled T) is similar to  $\triangle AFE$  as the three angles in each triangle are equal. The lengths of the edges of  $\triangle AFE$  are twice the lengths of the edges of  $\triangle ABC$  so  $\triangle AFE$  is an enlargement of  $\triangle ABC$  by a scale factor 2 with the centre of the enlargement at A. There are 4 copies of  $\triangle ABC$  in  $\triangle AFE$  so the area scale factor of the enlargement is 4.

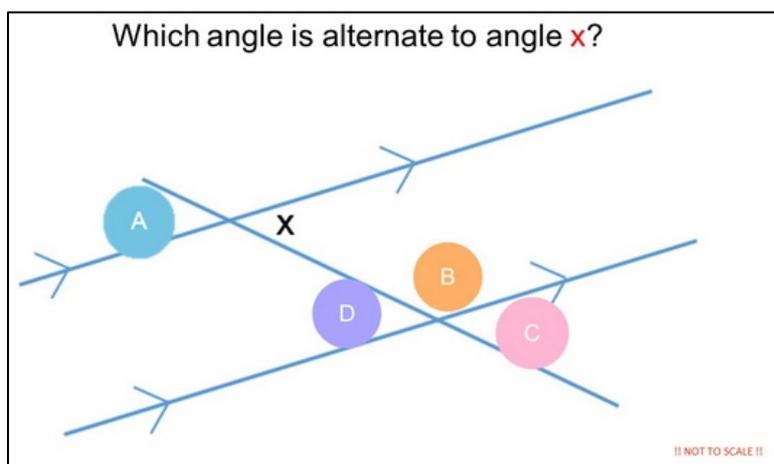
Many more geometrical facts can be seen from this diagram.

### DIAGNOSTIC ASSESSMENT

This should take about 5–10 minutes. It can be used before or after the lesson.

Write the question on the board, say to the class:

**“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.**



1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
3. Ask the class again to vote for the right answer by putting up 1, 2,

**3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

4. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

**D.** is the correct answer.

### **Common Misconceptions**

Wrong answers or guesses here mean that the learner does not know the word 'alternate'. Learning and remembering the mathematical words is not easy particularly if the English is an additional language.

This example gives the teacher a basis for reviewing the angle properties of parallels and, most importantly, helping the learners to remember the words.

A wall chart would be helpful (See page 4)

<https://diagnosticquestions.com>

## **Why do this activity?**

This activity can be used in different grades to engage learners in looking carefully at different geometrical properties illustrated in the diagram and talking about these properties. Learners can be asked to make a list of everything that they notice and then all the observations can be shared. This is a very open ended activity giving plenty of scope for learners to think mathematically, to develop their visualisation skills and to contribute to the class discussion. It can be revision or lead into introducing some new geometrical concepts.

## **Learning objectives**

In doing this activity students will have an opportunity for a review of the geometry of parallel lines, angles, triangles and polygons.

## **Generic competences**

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **develop visualization** and skill to interpret or create images to represent concepts and situations;

## **Suggestions for teaching**

**Resources:** *Tracing paper would be a useful resource here.*

*Worksheets downloaded and copied.*

Start the lesson with the Diagnostic Quiz. Then give out the worksheet and ask the learners to read it individually and to start to answer the questions, writing their answers in their workbooks. You may wish to give the HELP strip to learners who you expect to struggle, especially to those who got the answers wrong twice in the Diagnostic Quiz. The NEXT strip is for learners who finish the activity ahead of the rest of the class and who would benefit from an extra challenge.

Use the 1-2-4-more teaching strategy to give learners practice in reading information and answering mathematical questions that are skills that they need to develop so that they can do this in tests and in later life. At this stage you could give out tracing paper and suggest how it could be used. Learners should trace the diagram onto the tracing paper, then move the tracing paper around to identify copies of triangle T, equal angles etc. Learners might continue this for homework and make lists of what they discover.

In the next lesson or, if you want the learners to continue the work in class after about 10 minutes, ask the learners to work in pairs and to share ideas. According to the class and the time you want to spend on this activity, allow the pairs to work together for some time and then tell them to share their ideas with another pair. However crowded the classroom working in fours can usually be organised with pairs of learners turning around to work with a pair behind them.

Some pairs will have noticed properties that the other pair did not see, so after the groups of 4 have had time to share ideas, lead a class discussion based on what the learners have observed and learned. Finally summarize what has been learned.

The class could make a poster listing all the geometrical properties that the learners have noticed. You could challenge the learners to find more properties not on the list.

If you have an electronic whiteboard, then by projecting copies of this diagram you can mark in equal angles and other features in the diagram (for example outlining similar triangles of different sizes, parallelograms, hexagons etc). You could do this with a large copy of the diagram on the board and learners can do the same on tracing paper placed over a copy of the diagram.

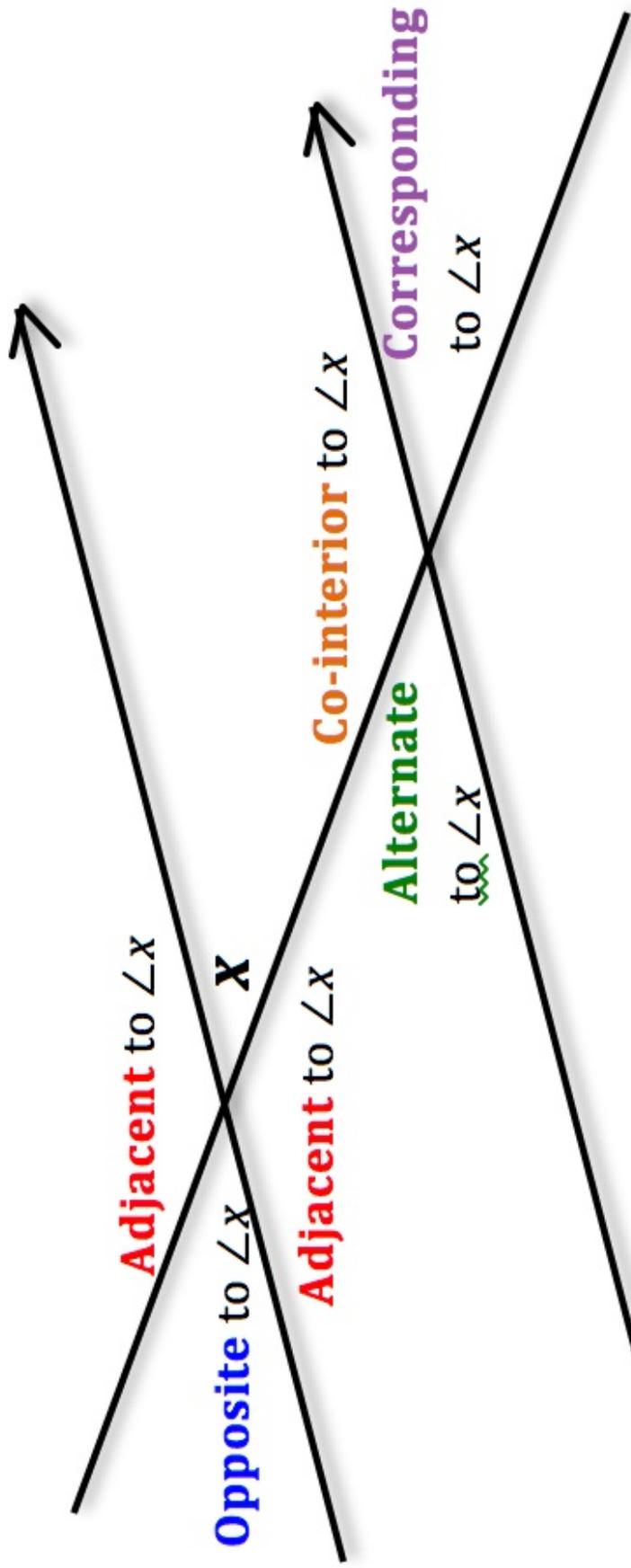
## Key Questions

- What shapes do you see?
- Which angles are equal? Why?
- What properties of the parallel lines do you see?
- Can you see similar triangles? How do you know they are similar?
- Can you see similar parallelograms? How do you know they are similar?

## Follow up

Tessellating Quadrilaterals <https://aiminghigh.aimssec.ac.za/years-7-12-tessellating-quadrilaterals/>

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. For resources for teaching A level mathematics see <a href="https://nrich.maths.org/12339">https://nrich.maths.org/12339</a> Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.				
	Lower Primary Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6



**PROPERTIES OF PARALLEL LINES AND ANGLES**