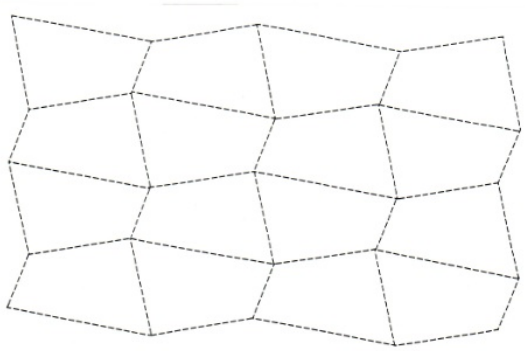
TESSELLATING QUADRILATERALS



We say that a shape tessellates if we can fit together copies of it to cover a flat surface without any overlaps or leaving any gaps and this covering can be extended in all directions.

The question here is "What can we find out about tessellating different quadrilaterals?"

It's quite easy to see how squares tessellate.

What about other types of quadrilateral?

Have a go at drawing some quadrilaterals, and finding ways to make them tessellate. You might like to cut a quadrilateral from scrap cardboard and use it to draw around again and again to make a tessellation.

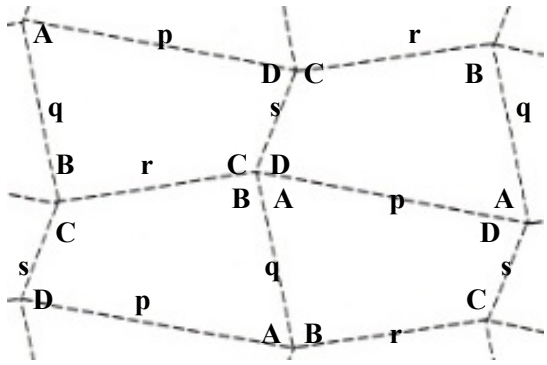
Think about different types of quadrilateral. For example, can you find a way to tessellate any parallelogram? What about a kite? Or a trapezium?

What do you notice about your tessellations?

Do all quadrilaterals tessellate? If your answer is no, give an example of a quadrilateral which doesn't tessellate. Can you explain why it doesn't tessellate?

If your answer is yes, can you explain why all quadrilaterals tessellate, and can you give a method which will produce a tessellation of any quadrilateral?

SOLUTION



All quadrilaterals tessellate.
The reason is that the angles of a quadrilateral add up to 360°. So in a tessellation the 4 angles of the quadrilateral come together at every vertex. In order to make the edges coincide, half the quadrilaterals are mirror images of the rest.

So to fit 4 copies of the quadrilateral together two have to be turned over then the 4 tessellate as in the diagram.

NOTES FOR TEACHERS

Why do this activity?

This is a practical activity that enables learners to draw their own quadrilaterals and make their own tessellations. It only requires scrap card. The fact that each learner can draw a quadrilateral with different angles from anyone else and all the learners will be able to draw a tessellation (with some experimentation and trial and improvement) leads to an obvious conjecture. The proof is simple and so this activity gives learners an opportunity to be creative and to make and prove their own conjectures.

Intended learning outcomes

Development of geometrical visualisation and practice in making and proving conjectures.

Possible approach

Ask the question “is it possible to make a tessellation pattern with all quadrilaterals whatever their shape?” Give learners some scrap card and scissors and ask them to draw a quadrilateral and then to see if they can use it to make a tessellation pattern. Ask them to draw their pattern on the board if they find the way to tessellate their quadrilateral. Some learners will draw squares, rectangles or rhombuses and the class can then move on to experimenting with other quadrilaterals.

Then ask the class “Do all quadrilaterals tessellate? If your answer is no, give an example of a quadrilateral which doesn't tessellate. Can you explain why it doesn't tessellate? If your answer is yes, can you explain why all quadrilaterals tessellate, and can you give a method which will produce a tessellation of any quadrilateral?”

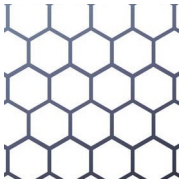
Give the learners time to work on this and to discuss their reasons with a partner then ask learners to come to the board and explain their working and proofs.

To make a connection between school maths and the outside world you might like to show some pictures of tiling used in buildings such as the examples given below. Ask learners to look out for different designs. They might take photos of tessellation designs that they see on their cellphones and the class could make a poster display. Make sure to include tiles other than polygons.

Key questions

What can you say about the angles of a quadrilateral.
How will you match that quadrilateral along that edge with another copy of itself?
Can you continue that pattern indefinitely in every direction.

Possible extension



We know that regular hexagons tessellate. Why is that?

Will all hexagons tessellate? Can you find non-regular hexagons that tessellate? Can you find a hexagon that does not tessellate? Under what conditions does a hexagon tessellate?

Possible support

You could start with Tessellating Triangles
https://aiminghigh.aimssec.ac.za/grades-7-to-12-tessellating-triangles/