



MATCHLESS

$$2x + 3y - 20$$

$$5x - 2y + 38$$

$$4x + 5y - 72$$

$$x - 4y + 108$$

$$3x - y + 39$$

Find values of x and y for which these five expressions all have the same value.

Do you have exactly the right amount of information or more information than you need?

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HELP

What happens if you take just two of the expressions?

NEXT

Make up a similar activity of your own?

Or, a bit more challenging, can you create an activity with an 'odd one out', that is where four expressions are equal for some specific x, y pair and there is one expression which does not equal the other four?

NOTES FOR TEACHERS

SOLUTION

We are told that $2x + 3y - 20 = 5x - 2y + 38 = 4x + 5y - 72 = x - 4y + 108 = 3x - y + 39$.

Just taking 2 of these expressions will give us an equation in x and y:

$$2x + 3y - 20 = 4x + 5y - 72$$

Taking $2x + 3y$ from both sides and adding 72 to both sides gives:

$$2x + 2y = 52 \quad (\text{Equation 1})$$

We need to find a second equation and solve the two equations simultaneously.

$$4x + 5y - 72 = 3x - y + 39$$

Taking $3x - y$ from both sides and adding 72 to both sides gives:

$$x + 6y = 111 \quad (\text{Equation 2})$$

Doubling Equation 2 and subtracting Equation 1 gives:

$$10y = 170$$

So $y = 17$ and $x = 9$.

Each of the expressions has the value 49.

The solution can be found using only 3 expressions. We are given more information than needed.

DIAGNOSTIC ASSESSMENT

This should take about 5–10 minutes.

Write the question on the board, say to the class:

“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.

- Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
- It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
- Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
- Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
- If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

MULTIPLE CHOICE

$$a = \frac{2}{3r} \quad \text{and} \quad 10 = \frac{a}{1-r}$$

Which of these is true?

A $\frac{2}{3r} = \frac{a}{1-r}$ **C** $10 = \frac{2}{3r(1-r)}$

B $10 = \frac{2(1-r)}{3r}$ **D** $10 = \frac{a}{1-\frac{2}{3r}}$

The correct answer is: C

Possible misconceptions:

A. Learners giving this answer are equating a with 10.

B. Here learners should have divided by $(1 - r)$ and not multiplied.

D. Here learners are mistakenly equating r with a .

<https://diagnosticquestions.com>

Why do this activity?

Tasks on simultaneous equations can become over-familiar, routine experiences for students. This type of problem causes a "stop and think" moment, requires some problem-solving ingenuity, and leads into thinking about how much information is needed.

Learning objectives

In doing this activity students will have an opportunity to:

- Get practice in reading and interpreting information given in a question;
- Get practice in tackling a non-standard question and in solving simultaneous linear equations.

Generic competences

In doing this activity students will have an opportunity to:

- get practice in problem solving;
- explore changing quantities.
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Suggestions for teaching

This is a good extension task for learners who are confident about solving simultaneous linear equations. It gives valuable practice in reading a question and thinking about how to use the information given. Give it to learners without any hints and ask them to work out for themselves how to tackle it.

Learners might make a start by substituting some arbitrary x, y values to get a feel for the problem and to realise that the five expressions don't generally take the same value. They will be unlikely to find the correct values of x and y just by 'trial and improvement' and they may realise for themselves that this is not the best method, so they will have to think of another strategy.

If learners have solved the problem by taking two pairs of expressions to form their two simultaneous equations then they might say that four expressions are needed to get the solution. In this case the teacher should ask if they can find the answer using fewer of the expressions.

Key questions

- What does the question tell you?
- What is the difference between an algebraic expression and an equation?
- Can you find any equations from the information given?
- What do you know about finding pairs of values?

Follow up

Symmetry <https://aiminghigh.aimssec.ac.za/symmetry/>

Graphical Triangle <https://aiminghigh.aimssec.ac.za/graphical-triangle/>

Intersections <https://aiminghigh.aimssec.ac.za/intersections/>