



FOR RICHER FOR POORER

In Durban the average salary is R160,000 and in Cape Town it is R180,000. Mdu moved from Cape Town to Durban.

Mdu has made both cities richer because the average salary has gone up in both due to his move. Can you explain how that could happen?

Describe what is needed for one person, by moving from one city to another, to:

- increase the average income in both cities;
- decrease the average income in both cities or
- increase one whilst decreasing the other.

In what circumstances could Mdu, moving from one place to another, reduce the average income in one place and double the average income in the other?

HELP

Try this simpler example first. It has all the essential features of the example about average earnings and moving from city to city.

Investigate what happens to the mean when different members move from one group to the other.

Group 1: ages 2, 8, 10, 14 and 16 (mean 10)

Group 2: ages 5, 11, 12,13, and 19 (mean 12).

NEXT

The open nature of this activity offers opportunities to think of other effects and test whether they are possible. For example:

- Is there a limit to the number of people who can leave Cape Town and have the same effect?
- Is it possible to double the income in one place whilst halving in the other?

You may also ask some questions of your own applying similar thinking in different situations like the example about mean ages in the HELP section.

NOTES FOR TEACHERS

SOLUTION

a) **To increase the average income in both cities**, someone with an income between R160,000 and R180,000 (but not exactly either) needs to move from Cape Town to Durban. So if Mdu's income was between the two amounts then he has increased the income of both cities. In Cape Town his income was below average so when he left the average there increased. In Durban his income is above average so when he started work there the average increased.

To change the average income of one and not change the other by moving either way, Mdu must have exactly the average of one of the two places.

b) **To decrease the average income in both cities**, Mdu must again have an income between the averages for the two cities so he is an above average earner in Durban and he must move from Durban to Cape Town, That way he will decrease the average of both cities.

c) **Increasing one while decreasing the other.** There are four ways of this happening

Mdu earns over R180,000 and moves from Cape Town to Durban. Then Cape Town's average goes down and Durban's goes up.

Mdu earns over R180,000 and moves from Durban to Cape Town. Then Cape Town's average goes up and Durban's goes down.

Mdu earns less than R160,000 and moves from Cape Town To Durban. Then Cape Town's average goes up and Durban's goes down.

Mdu earns less than R160,000 and moves from Durban to Cape Town. Then Cape Town's average goes down and Durban's goes up.

To reduce the average earnings in A and double it in B a person with a salary above the average for A must move from A to B.

To calculate what the salary needs to be introduce some symbols and equations.

Let b be the average income in B, n the number of people with jobs in B, s the salary of the person moving from A to B.

Then the total of all the salaries before the move is nb and the total after is $nb + s$.

After the move there are $(n + 1)$ people with jobs in B so the average salary is $(nb + s)/(n + 1)$

Suppose this average salary has doubled then $(nb + s)/(n + 1) = 2b$.

Simplifying this expression gives $s = 2b(n + 1) - nb = b(n + 2)$.

Suppose B is a village where 7 people have jobs and the average earnings are R50,000.

Then using the formula $n=7$ and $b=50000$ so for the income in B to double

$$s = 50000(7 + 2) = 450\,000$$

Diagnostic Assessment This should take about 5–10 minutes.

Write the question on the board, say to the class:

“Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D”.

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

2. It is important for learners to explain the reason for their answer to clarify their own thinking and practise communication.

3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**

5. As the concept is needed for the lesson to follow explain the right answer or give a remedial task.

B. is the correct answer.

Common Misconceptions

A. This is the total not the mean of the numbers;

C. The answer is 6.666... so this is not correctly rounded;

D. May have tried to add the numbers and got the wrong answer.

<https://diagnosticquestions.com>

Work out the mean
5, 3, 7, 7, 7, 11
Correct to 1 decimal place

a) 40
b) 6.7
c) 6.6
d) 41

Why do this activity?

This activity may lead to a deeper understanding of averages. The unexpected result may lead learners to ask what else might be possible. Encouraging learners to ask questions. Key skills that are important for learners to develop are involved here so encourage learners to look for different possibilities and to organize their work in a systematic way so as to be able to include all possibilities and to draw conclusions

Intended learning outcomes

- a deeper understanding of averages.

Generic competences

In doing this activity students will have an opportunity to:

- interpret information and solve problems;
- analyze, reason and record ideas effectively.

Suggestions for teaching

Start with the diagnostic assessment then introduce this simpler question. Write this table on the board:

Abe	Buk	Chris	Dudu	Erin
2	8	10	14	16

The table shows the ages of 5 children. What is their average (mean) age?

If Abe leaves the group what happens to the average?

If Chris leaves the group instead what happens to the average?

If Erin leaves the group instead what happens to the average?

From this the learners should realise that the mean can stay the same or go up or down depending on the age of the child who leaves. This is the focus of the main problem.

Introduce the main challenge, either by writing it on the board or photocopying the top section of page 1 to make worksheets. The first part should be straightforward so most learners should be able to devote time to investigating the different effects that that moves between cities have on the average income.

Let the learners work in pairs and, later in the lesson, invite learners to present what they have found out to the rest of the class. This may offer an opportunity to reflect on the value of approaching the work in a systematic way.

Key questions

- In what ways can the means change?
- What are the possibilities?
- What are the variables that can be altered?

Follow up

For Younger for Older (a similar and simpler problem)

<https://aiminghigh.aimssec.ac.za/years-5-9-for-younger-for-older/>

M, M and M <https://aiminghigh.aimssec.ac.za/years-8-11-m-m-and-m/>

Mean, Median, Mode and Range

<https://aiminghigh.aimssec.ac.za/years-8-9-mean-median-mode-and-range/>