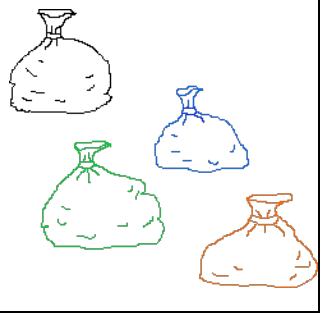
MONEY BAGS



Ram had 15 coins, **all the same kind**, and he put them into 4 bags.
He labelled each bag with the number of coins inside it.
He could then pay any sum of money from 1 coin to 15 coins with one or more of the bags and without opening any of the bags.

How could he do that?

HELP

Try this for yourself. Use stones or counters for the coins or make them from scrap paper. Experiment by putting the coins in 4 piles or bags and rearrange them until you find the solution. Keep a record of what you have tried to save wasting time by trying the same thing over and over again.

NEXT

Suppose you had another bag and some more coins. What would be the amount to put in the fifth bag so you could make all the amounts 16, 17, 18, …. and as many as possible?

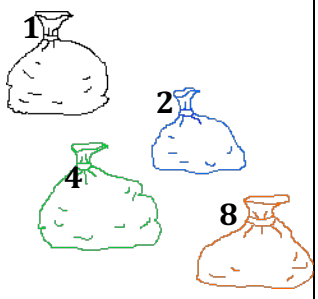
NOTES FOR TEACHERS

SOLUTION

You must be able to pay any sum of money from 1 coin to 15 coins without opening any of the bags, so one bag must have 1 coin in it and another bag must have 2 coins in it. With these two bags you can pay 1 coin, 2 coins or 1+2=3 coins but not more.

So you must have a bag with 4 coins in it. With 1, 2 or 3 of these 3 bags you can pay 1, 2, 3, 4, 5, 6 or 7 coins but not more.

So you must have a bag with 8 coins in it. This uses all 15 coins.



You can pay the following amounts:	**Another way of recording the solution:**				
	Bags used	1	2	4	8
	Value				
1	1	X			
2	2		X		
3 = 1 + 2	3	X	X		
4	4			X	
5 = 1+4	5	X		X	
6 = 2+ 4	6		X	X	
7 = 1+2+4	7	X	X	X	
8	8				X
9 = 1+8	9	X			X
10 = 2+8	10		X		X
11 = 1+2+8	11	X	X		X
12 = 4+8	12			X	X
13 = 1+4+8	13	X		X	X
14 = 2+4+8	14		X	X	X
15 = 1+2+4+8	15	X	X	X	X

BINARY NUMBERS	**DECIMAL NUMBERS**
Numbers written as powers of 2	**Numbers written as powers of 10**
1	1
10	2
11	3
100	4
101	5
110	6
111	7
1000	8
1001	9
1010	10
1011	11
1100	12
1101	13
1110	14
1111	15

Why do this activity?

This activity offers a challenge that needs a systematic approach and mathematical thinking but only requires knowledge of simple counting and addition of small whole numbers. The activity also leads naturally to thinking about whether the pattern can be continued for larger amounts. The solution might be linked to binary numbers, as used by computers, and then used to reinforce an understanding of place value. The activity lends itself to a focus on different ways of recording.

Learning objectives

In doing this activity students will have an opportunity to:

- practice simple addition;
- develop problem solving skills;
- develop confidence in spotting patterns and solving problems and see the value of working systematically – in this case looking for a solution for 1, 2, 3, 4, 5 … in turn;
- relate the pattern to binary numbers and gain a deeper understanding of place value.

Generic competences

In doing this activity students will have an opportunity to:

- work systematically;
- develop logical thinking:

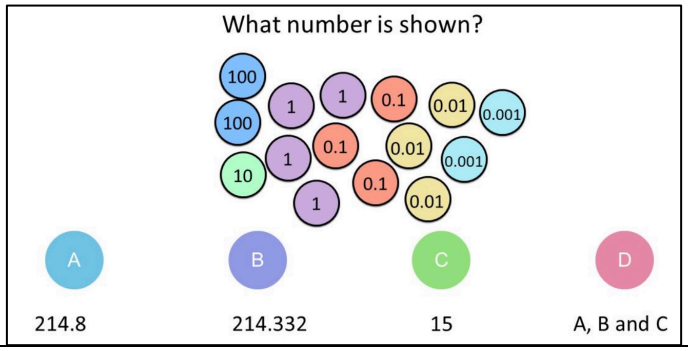
- apply knowledge and skills;

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. It is important for learners to explain the reason for their answers to develop their own communication skills and to help other learners to hear different ways of thinking about the question.
4. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
5. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**
6. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

B. is the correct answer.

Common Misconceptions



A. These learners have seen the 2 hundreds, 1 ten and 4 units and understood that the number is 214 point something but they are confused about the decimal part and just counted the 8 counters as if they were all 0.1

C. These learners have counted 15 counters and given the answer 15 without understanding that the counters have different values.

D. These learners are probably just guessing.

https://diagnosticquestions.com

Suggestions for teaching

Start with the diagnostic question. This may give learners the idea that the bags will have different values like the counters in the diagnostic question.

Learners may not at first understand what the problem is about. So you could begin by reversing the situation. Draw two bags on the board and label them 1 coin and 3 coins and emphasise that the coins are all the same kind. Invite learners to suggest the amounts you could pay using these bags if you are not allowed to open either of them [*Answer: 1, 3 and 4 coins*]. Introduce another bag which contains 5 coins to go with the first two. This time, ask the learners which amounts of money they ***would not*** be able to make using these bags [*Answer: 2, 7, and anything of 10 coins and above*]. You could give them time in pairs to work on this, perhaps using a showboard to make jottings.

Invite pairs to share their solutions and highlight those that have used a systematic approach, for example by starting with the smallest amount of money and working up in steps of a coin at a time. You can then introduce the problem as it stands. Give pairs chance to work together before once again sharing their ideas. Before they give the answer, ask learners to explain how they went about finding the solution.

The whole class could then check the solution offered by trying to make all the different amounts from 1 coin to 15 coins. You may also wish to share the different ways of recording used by various learners. You could discuss the advantages of each method and it would be worth asking whether learners would change their chosen method if, for example, they were told someone else was going to read it. This gives you an opportunity to highlight the fact that we might record differently, depending on what the recording is for and who is going to see it.

Key questions

How will you make a payment of 1 coin?
How will you make a payment of 2 coins? ...
How will you make sure you can pay for all the amounts from 1 coin to 15 coins?
Have you checked them all?

FOLLOW UP

Which Scripts https://aiminghigh.aimssec.ac.za/which-scripts/

Balance Power https://aiminghigh.aimssec.ac.za/balance-power/

Writ Large https://aiminghigh.aimssec.ac.za/writ-large/

Spin High Spin Low https://aiminghigh.aimssec.ac.za/spin-high-or-low/

Find the Numbers https://aiminghigh.aimssec.ac.za/find-the-numbers/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/MathsToys/videos
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13