AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

MINIMISING OUTPUT

The smallest number that can be divided exactly by 2, 3 and 4 is 12.

What is the smallest number that can be divided exactly by the numbers 2, 3, 4, 5 and 6?

What is the smallest number that can be divided exactly by the numbers 2, 3, 4, 5, 6, 7, 8 and 9?

Help

Start with "What is the smallest number that divides exactly by 2, 3, 4, and 5?"
Is it $120 = 2 \times 3 \times 4 \times 5$? Why or why not?

How did you find the right answer?

Then ask yourself: "What is the smallest number that divides exactly by 2, 3, 4, 5 and 6?"
Does that change the answer? Why or why not?

Now include 7 in the list. Does that change the answer?

The question to be answered was "What is the smallest number that can be divided exactly by the numbers 2, 3, 4, 5, 6, 7, 8 and 9?" It is easier to get the final solution in stages by including **one more number in the list at each stage**. So can you keep trying and get the final answer?

Extension

The list of numbers in the question can be extended to include 10, then extended to include 10 and 11 and so on…

Carry on this process to see if you can find any patterns.

NOTES FOR TEACHERS

SOLUTION

Notice that 12 = 3 x 4 and 4 is divisible by 2. So to find the smallest number that can be divided exactly by 2, 3 and 4 we write the product of PRIME NUMBERS: $2^2 \times 3 = 12$.

Notice that 12 is also divisible by 6. So, to find the smallest number that can be divided exactly by the numbers 2, 3, 4, 5 and 6 we only have to multiply by 5 to find the solution which is 5 times 12, that is 60. It is instructive to see 60 written as a product of primes: $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

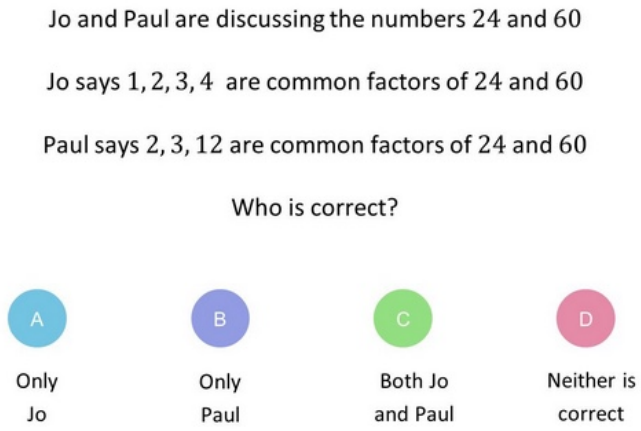
To find the smallest number divisible by 2, 3, 4, 5, 6, 7 we multiply 60 by 7 to give 420.

To find the smallest number divisible by 2, 3, 4, 5, 6, 7 and 8 we only have to multiply by 2 as 420 is divisible by 2 and 4 but not by 8. This gives 420 x 2 = 840.

To find the smallest number divisible by 2, 3, 4, 5, 6, 7, 8 and 9, we only have to multiply by 3 as 840 is divisible by 3 but not by 9. This gives the solution : $2520 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2^3 \times 3^2 \times 5 \times 7$

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



The correct answer is **C**
Factors of 24: 1, 2, 3, 4, 6, 8, 12 and 24
Factors of 60: 1, 2, 3, 4, 5, 6, 10, 12 15, 20, 30 and 60
The common factors are: 1, 2, 3, 4, 6, 12
A and **B**. Perhaps students giving these answers misread the question.
D. It seems that students giving this answer do not understand common factors.

https://diagnosticquestions.com

Why do this activity?

This activity engages learners in thinking about factors and multiples, understanding the relationship between division and multiplication. It gives an opportunity to use the language of multiplication, division, factors and multiples. The question is suitable for different age groups as it can be answered without referring to primes or exponents.

Learning objectives
In doing this activity students will have an opportunity to deepen their understanding of multiples.

Generic competences
In doing this activity students will have an opportunity to analyze, reason and record ideas effectively.

Suggestions for teaching
Start with the Diagnostic Quiz. You might then ask learners to multiply the numbers 2, 3 and 4 and they will get the answer 24. Then ask if 24 can be divided exactly by 2, 3 and 4 and discuss why it is that the number 12 can also be divided exactly by 2, 3 and 4.

It is your decision as to whether you engage the class in talking about prime numbers and in writing the numbers as products of primes as the question can be answered without referring to primes.

Then give the learners time to answer the question "What is the smallest number that can be divided exactly by the numbers 2, 3, 4, 5 and 6?" perhaps working in pairs. If they struggle suggest that they consider only 1, 2, 3, 4, and 5 before they go on to also include 6. Give the learners time to work this out for themselves. Then have a whole class discussion asking the learners to explain how they found their answers.

Finally ask them to answer the last part of the question. Suggest that they should take this one step at a time, including only 7, then 8 and finally 9. This could be a homework.

Key questions
- What does it mean to say that one number can be divided exactly by another number?
- What does it mean to say one number is a factor of another? Give me an example.
- What does it mean to say that one number is a multiple of another? Give me an example.
- What answer do you get if you multiply those numbers together?
- Is there a smaller number that is divisible exactly by all those numbers? How can you be sure?
- Can you explain how you found your answer?

Follow up
Patterns of multiples: https://aiminghigh.aimssec.ac.za/years-5-8-multiple-patterns/
Finding prime numbers: https://aiminghigh.aimssec.ac.za/years-6-9-prime-sieve/
Multiples and cog wheels: https://aiminghigh.aimssec.ac.za/years-6-10-turning-cogs/
Special properties of multiples of 37: https://aiminghigh.aimssec.ac.za/years-4-to-7-magic-of-37/
Multiply 3 numbers for the volume of a cuboid: https://aiminghigh.aimssec.ac.za/years-6-to-8-same-volume/
Patterns of certain multiples of 9: https://aiminghigh.aimssec.ac.za/years-6-9-times-nine/
A game for two players and challenges for the whole class:
https://aiminghigh.aimssec.ac.za/years-6-12-factors-and-multiples-game/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**