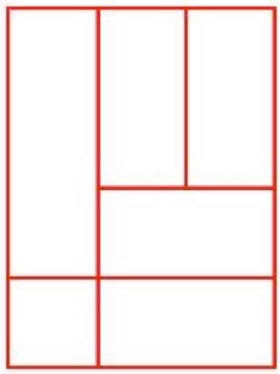
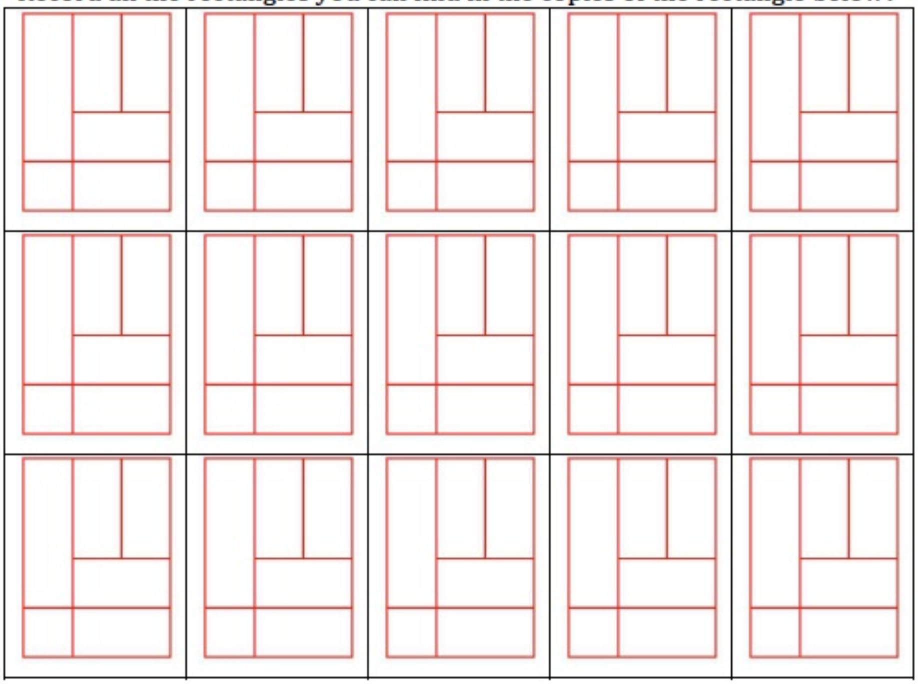
IS A SQUARE A RECTANGLE



How many rectangles can you find in this shape?
Which of your rectangles are the same (congruent)?
Which of your rectangles are similar (enlargements of each other)?
Record all the rectangles you can find in the copies of the rectangle below?



HELP

First shade all the rectangles that measure 2 units by 1 unit, each on a **different** copy of the diagram. Great, that's a good start.

Now shade all the squares. How many rectangles have you found now? You are well on your way.

Can you see any 3 by 1 rectangles? Good, now shade them and then look for more possibilities.

NEXT

Write down your answer to the question 'Is a square a rectangle?' and give a full explanation.

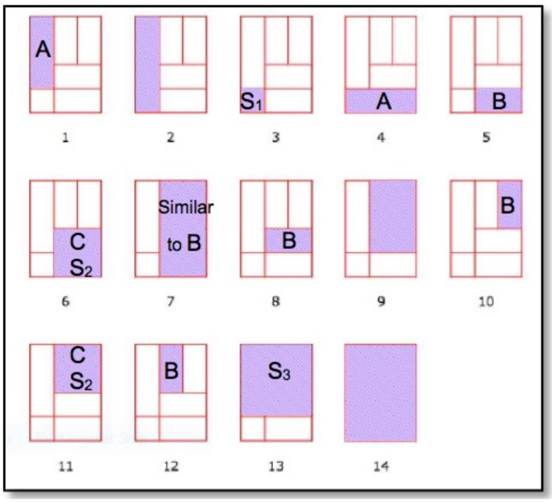
Describe the transformations that map each rectangle onto other congruent rectangles.

Describe the transformations that map each rectangle onto other similar rectangles. Can you do this without using reflections?

Draw your own diagram and make up a similar problem.

NOTES FOR TEACHERS

SOLUTION



1) Rectangles 2, 9 and 14 are not congruent or similar to any rectangles in the diagram.

CONGRUENT RECTANGLES

2)Rectangles 1 and 4, labelled A, are congruent (the same as each other) (1 by 3).

Rectangle 1 maps to coincide with rectangle 4 by

a rotation of 90º clockwise about its bottom left hand corner

3)Rectangles 5, 8, 10 and 12 labelled B, are congruent (1 by 2).

4)Rectangles 6 and 11 (which are squares) are congruent (2 by 2).

SIMILAR RECTANGLES

5) Rectangles 5, 8, 10 and 12 (1 by 2) are similar to rectangle 7 (2 by 4) scale factor 2.

SQUARES 3, 6, 11 and 13 ARE ALL SIMILAR

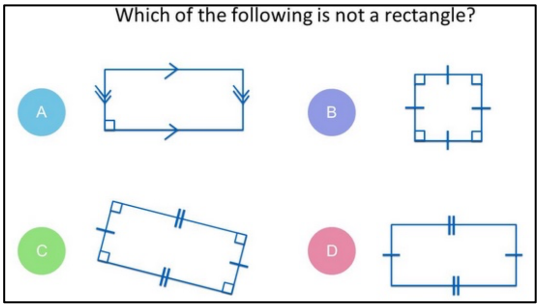
6) 3 (1 by 1) and 13 (3 by 3) *are squares* similar to each other with scale factor 3.

7) 3 (1 by 1) is similar to squares 6 and 11 with scale factor 2.

8) 13 (3 by 3) is similar to squares 6 and 11 with scale factor 1.5.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. It is important for learners to explain the reason for their answer to practise mathematical thinking and communication.
4. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
5. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.**

The correct answer is **D.**. This is a parallelogram. There is no reason to assume that the angles are right angles so it is not a rectangle.

Common Misconceptions: Learners giving answers A, B or C may have missed the 'not' in the question.
A. Learners do not see that the parallel lines make all 4 angles equal to 90º so this is a rectangle.
B. Common misconception! This satisfies all the conditions for a rectangle so it is a rectangle. As it has 4 equal sides it is also known as a square.
C. This is a rectangle. Is this a language issue? Did the learner understand the question?
https://diagnosticquestions.com

Why do this activity?

This activity gives learners the opportunity to practise mathematical thinking and visualization. It is suitable for learners from age 10 upwards. The definitions of similarity and congruence can be made clear through this activity because spotting similarity and congruence between rectangles is easier than doing it for triangles. Learners will experience having to decide whether two shapes are the same in all respects (congruent) or whether they are similar (enlargements of each other) without having to know the technical language. You can use the activity to illustrate the idea that similarity and enlargement are essentially the same property described in different mathematical language. Scale is one of the major concepts in mathematics and this activity gives practice in working out scale factors in very simple cases. It gives learners a challenge to find all possible solutions and they need to work systematically and think of a clear explanation as to why there are no more than 14 solutions.

Intended learning outcomes

In doing this activity students will have an opportunity to:

- gain a preliminary understanding of, or to review, the concepts of similarity and congruence.
- practise and develop the skills of working systematically and problem solving.
- deepen understanding of scaling, similarity and enlargement.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically**, reason logically and give explanations and proofs;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- interpret and **solve problems** in a variety of situations;
- by the 1-2-4-more teaching strategy to:
  - work and **learn indepe**ndently and prepare for lifelong learning;
  - **work in a team**:
    - co-operate and collaborate/work with a partner or group
    - have empathy with others, listen to different points of view
    - develop leadership qualities;
- **communicate** in writing, speaking and listening according to the audience:
  - exchange ideas, criticise, and present information and ideas to others
  - analyze, reason and record ideas effectively.

Suggestions for teaching

For older learners this could be a warm-up task or lesson starter that is ready on the board so learners get busy as soon as they sit down in class. It provides a challenge and an opportunity for revision that need not take up much lesson time, especially for upper secondary school students.

ENCOURAGE LEARNERS TO READ THE PROBLEM AND, IN PAIRS, WORK OUT FOR THEMSELVES WHAT THEY NEED TO DO.

Very often teachers say that learners can do the mathematics in class but do badly in tests. One reason for this could be that learners find it difficult to read the question and work out for themselves what they have to do. Teachers often give too much help by reading the problem and explaining what the learners must do, perhaps using their home language. You might explain to the learners that they need to get used to reading questions for themselves. Give them this list of guidelines for how to read the problem and what to ask themselves.

1. Read it slowly. Read it bit by bit
2. "Do I understand all the words?"   If not look them up, ask a friend or ask the teacher.
3. "What am I supposed to do?"
4. "Do I know any maths that could help?"

REPORT BACK See page 4 for templates to photocopy for displaying the solutions. You could prepare a wall chart in advance and invite learners to come and shade another rectangle as soon as they find one that is not already there. If the class between them don't find all 14 solutions give the learners more time to work on the challenge, not necessarily in lesson time. You could call a halt and move on to something else but invite learners to come back the next day with more solutions. Such challenges can be left for a week or two with repeated encouragement and hints from the teacher so that learners have the incentive to keep trying.

REVIEW When all 14 solutions have been found an the learners have decided which are congruent and which are similar have a class discussion so that you ensure that all the learners understand the concepts of congruence and of similarity and scale factors. You should number the rectangles so that learners can refer to them.

Use mini-whiteboards (showboards) to ensure that you get feedback from EVERY learner. Showboards are easily made by laminating an ordinary sheet of paper and are most useful if plain on one side and with a grid on the other side. If the class identifies that a certain pair of rectangles are similar then you could ask them simply to write the scale factor on their mini-whiteboard. Alternatively you could give them a scale factor (for example 1.5) and then ask them to write down and show you the number-labels of the pair of similar rectangles with that scale factor between them.

Key questions

- What do we know about rectangles, how do we recognize one when we see one?
- OK a rectangle is a quadrilateral with 4 angles of 90^o does it follow that the opposite edges must be parallel? If so why?
- If you have a quadrilateral with 4 angles of 90^o (rectangle), could this be true and at

the same time could all 4 edges be the same length (a square)?

- Can you spot two rectangles that are congruent? How do you know?
- Can you spot two rectangles that are similar? How do you know?
- You say those two rectangles are similar. What are their edge lengths? What is the scale factor?
- Can you draw a diagram on squared paper (or cut out) a pair of similar rectangles and place them so that you can show lines joining corresponding points and the centre of enlargement?

Follow up

Properties of parallelograms
https://aiminghigh.aimssec.ac.za/properties-of-parallelograms/
Properties of quadrilaterals
https://aiminghigh.aimssec.ac.za/properties-of-quadrilaterals/

Go to the **AIMSSEC AIMING HIGH** website for lesson ideas, solutions and curriculum links: http://aiminghigh.aimssec.ac.za
Subscribe to the **MATHS TOYS YouTube Channel**
https://www.youtube.com/c/MathsToys/videos
Download the whole AIMSSEC collection of resources to use offline with the **AIMSSEC App** see https://aimssec.app or find it on Google Play.

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13