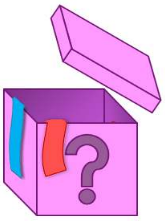
IN A BOX



This is a game for two players. You can use strips of coloured paper in a box or in a paper bag instead of ribbons in a box. The important rule is that each player takes two ribbons without being able to see their colour.

Najwa and Zuki play this game.
They put two red and four blue ribbons in a box. They pull out two ribbons without looking at the colours.

Najwa wins if both ribbons are the **same colour.**
Zuki wins if the two ribbons are **different colours.**

Play the game with a friend.

Who wins more often? Is this a fair game? If not, why is it unfair?

Make a list of all the different possible events that can happen each time you play this game, for example (red and red) …

Now suppose that you change the game so that there are two red and four blue ribbons in the box?

Is this a fair game? Why or why not?

If the game is not fair, how would you change the game to make it fair?

Is there any skill in this game or is it just luck?

HELP

Call the ribbons R1, R2, B1, B2, B3 and 43. Make a list of all the possible pairs of ribbons that you can take out. For example R1-B3 means you take R1 first then B3 and B3-R1 means you take B3 first then R1.

Count the number of draws for which Najwa wins, and the number of draws for which Zuki wins? Is this fair? What assumptions are you making?

NEXT

Make up a similar game that is fair.
Draw a tree diagram for this game.

NOTES FOR TEACHERS

SOLUTION

Although the same colour, the ribbons are distinct from each other so we label them R1. R2, B1, B2, B3 and B4.

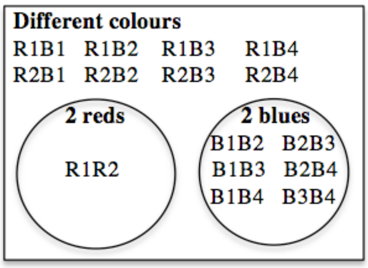
Method 1
Listing all choices without considering order

The different ways of choosing 2 ribbons are listed below if the order they are chosen is not considered:

R1 R2				
R1 B1	R2 B1			
R1 B2	R2 B2	B1 B2		
R1 B3	R2 B3	B1 B3	B2 B3	
R1 B4	R2 B4	B1 B4	B2 B4	B3 B4

Of the 15 possibilities listed above one is a choice of 2 red ribbons and 6 are choices of 2 blue ribbons. So the probability of 2 reds is 1/15 and the probability of 2 blues is 6/15 making the probability of two the same colour and Najwa winning to be 7/15. There are 8 ways that Zuki can win. So it is not a fair game

SAMPLE SPACE DIAGRAM



The sample space shows 3 distinct sets.

Method 2
Listing all choices taking order into account

Or alternatively we can list the possibilities according to the order the ribbons are chosen, one set above and to the right of the diagonal line and the other set below and to the left.

R1 R2	R2 R1	B1 R1	B2 R1	B3 R1	B4 R1
R1 B1	R2 B1	B1 R2	B2 R2	B3 R2	B4 R2
R1 B2	R2 B2	B1 B2	B2 B1	B3 B1	B4 B1
R1 B3	R2 B3	B1 B3	B2 B3	B3 B2	B4 B2
R1 B4	R2 B4	B1 B4	B2 B4	B3 B4	B4 B3

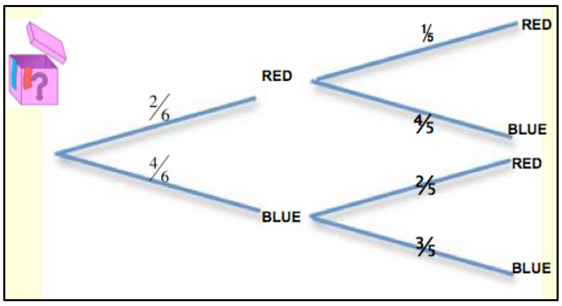
To list all 30 possibilities we can use a contingency table

	1st red	1st blue	
2nd red	2	8	10
2nd blue	8	12	20
	10	20	30

The probability of 2 reds is 2/30 = 1/15
The probability of 2 blues is 12/30 = 6/15

So the probability of Najwa winning is 7/15 and the probability of Zuki winning is 8/15

Method 3 - using a tree diagram



(Red, Red) = 2/6 × 1/5 = 1/15

P(Red, Blue) = 2/6 × 4/5 = 4/15
P(Blue, Red) = 4/6 × 2/5 = 4/15

P(Blue, Blue) = 4/6 × 3/5 = 6/15 = 2/5

The probability both ribbons are the same colour is 1/15 + 6/15 = 7/15.
The probability the ribbons are different colours is 4/15 + 4/15 = 8/15.
This is not a fair game as Zuki has a better chance of winning with 2 ribbons of ***different*** colours.

What happens if there are the same number of ribbons of each colour?

Suppose for example there are 4 of each colour.
P(RR) + P(BB) = (4/8 × 3/7) + (4/8 × 3/7) = 24/56 < ½ so this is not a fair game.
Check P(RB) + P(BR) = (4/8 × 4/7) + (4/8 × 4/7) = 32/56 = 1 – 24/56

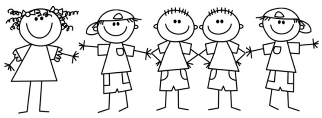
Change the game to 2 ribbons of each colour.
Najwa wins if they pick 2 reds and Zuki if it is 2 blues.
P(RR) = 2/4 x 1/3 = 1/6
P(B,B) = 2/4 x 1/3 =1/6
This is the same probability so it is a fair game.
It will be a draw if the ribbons are different colours.

What if there are 3 red ribbons and 1 blue ribbon?
P(RR) = 3/4 × 2/3 = ½ so this is a fair game
Check P(RB) + P(BR) = (3/4 × 1/3) + (1/4 × 3/3) = ¼ + ¼ = ½

Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



There are 4 boys and 1 girl in a group.

(1) How many ways can two of them be chosen to do a job and
(2) how many ways can 2 boys be chosen.
(The order of choice does not matter)

A. 10 and 5 B. 4 and 6 C.4 and 10 D. 10 and 6

2. Notice how the learners responded. Ask them all to explain why they gave their answer and **DO NOT** say whether it is right or wrong but simply thank the learner for giving the answer.
3. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask them to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

The correct answer is D

(1) The 10 choices are: GB_1, GB_2, GB_3, GB_4, B_1B_2, B_1B_3, B_1B_4, B_2B_3, B_2B_4, B_3B_4.
(2) Two boys can be chosen in 6 ways.

Some of the other answers show that the learner knows about counting pairs but did nut fully understand what the question was asking. This may be a language comprehension problem rather than a maths difficulty.
https://diagnosticquestions.com

Why do this activity?

This game can be played simply for fun and the learners can be encouraged to work out whether it is a fair game or not. Alternatively you could tell them it is not a fair game and the challenge could be to discover why. This offers opportunities to use different methods. Teachers can select a particular method or, with older learners, encourage them to choose their own method without advice from the teacher. The activity provides practice in developing the skill of working systematically to list the possibilities.

Learning objectives

Younger learners will have an opportunity to:

- play a game and learn how to count the number of outcomes;
- experiment to investigate whether a game is fair or unfair.

Older learners will have an opportunity to:

- play a game, count the number of outcomes and work out the probability of different events;
- see a variety of methods of solving one problem and compare them;

- invent their own games distinguishing between fair games and unfair games.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically and systematically,** reason logically and give explanations;
- **think flexibly**, be creative and innovative and apply knowledge and skills;
- interpret and **solve problems** in a variety of situations;
- collaborate and work with a partner or group and play competitively showing respect and **consideration for others.**

Suggestions for teaching

Let the learners play the game a few times. Ask:
"Is this a fair game? How can we be sure?"

The class should work in pairs or small groups trying to decide whether the game is fair and developing an argument to justify their conjectures.

About half-way through the lesson ask different groups to explain their methods. It is possible that different groups will come to different conclusions about whether the game is fair or unfair, but they should give reasons. Different groups might suggest and explain different methods.

This is sufficient for younger learners.

For older learners: the class should discuss the merits of different arguments and representations. This may be an appropriate point to highlight the benefits of different systematic methods for listing all possibilities, using sample space diagrams and, if learners have met them before, tree diagrams.

Some 'high flyer' students can look for fair games while other students work on the original problem.

Alternatively, finding a fair game can become a class activity and learners might help to create a class list of all distinct starting points for the game (for example, four ribbons can be either 1 red and 3 blue, 2 red and 2 blue, 3 red and 1 blue or 4 red and 0 blue).

The alternatives are written on the board for 2, 3, 4, 5,…ribbons.
Distribute the task of checking which combinations are fair and record them on the board as the learners decide and the class agrees.

There are not many solutions that give fair games. If learners are to notice a pattern amongst the combinations that are fair they may need to consider up to a total of 16 ribbons, spend some time conjecturing about more than 16 ribbons and test conjectures.

Key questions

- How can you decide if the game is fair?
- How many goes do you think we need to be confident of the likelihood of winning?
- Are there efficient systems for recording the different possible combinations?
- Can you justify your conclusions?

Follow up

Spin High or Low https://aiminghigh.aimssec.ac.za/years-4-7-spin-high-or-low/
Two Aces https://aiminghigh.aimssec.ac.za/years-10-to-12-two-aces/
In the Bag https://aiminghigh.aimssec.ac.za/years-9-to-12-in-the-bag/
Lucky Numbers https://aiminghigh.aimssec.ac.za/grades-7-to-12-lucky-numbers/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6