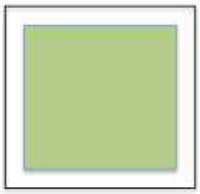
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

SQUARE FENCE

A fence is built around a square field. Suppose another fence is built exactly one metre longer so the path between the two fences is the same width along the edges of the field.



1. How wide would this path be?
2. Would a mouse be able to run along this path?
3. Could a farmer drive a herd of cows along the path between the fences?
4. Suppose you were the farmer and you wanted to make the path a certain width how would you work out how much fencing to buy?

Help

Draw a square. Cut very narrow strips of scrap card and make a fence around the square field that you have drawn. Then to make a longer square fence with a path around the field. Think about where the extra fence goes to make the longer fence, and how this is connected to the width of the path.

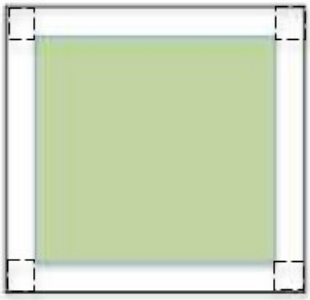
Next

Try the 'Not-so-Square Fence' https://aiminghigh.aimssec.ac.za/grades-9-to-10-not-so-square-fence/ on the AIMING HIGHER Teacher Network. This is essentially the same problem but the path is a constant width everywhere and so the fence has to be rounded at the corners of the field.

NOTES FOR TEACHERS

SOLUTION

Note: This is the first of a sequence of three problems: Square Fence; Not-so-square Fence and Belt Around the Earth. Each one leads to the next and doing them in sequence will help learners to solve the harder problems.



This is a question about perimeter. The key to a quick and easy solution here is to understand that the edges of the fence parallel to the edges of the field will always be the same WHATEVER THE SIZE OF THE FIELD and that the extra fence is only needed at the corners.

Method 1 If the width of the path is w, and you think about the extra fence at each corner, it forms two edges of length w at right angles. This means that the extra fence makes a square and it has length 8w metres for any sized field.
We know that the extra length of fence is 1 metre so the width of the path is one eighth of a metre, that is 12.5 centimetres.
The path is too narrow for cows but a mouse could easily run along it.

Method 2 This algebraic method is equivalent to the mathematical reasoning. Let the width of the side of the square be a and the width of the path be w then

$4(a + 2w) = 4a + 1$

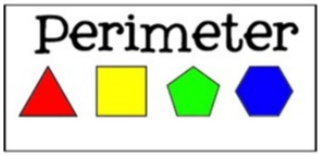
$4a + 8w = 4a + 1$

$w = \frac{1}{8} = 0.125$ m $= 12.5$ cm

Diagnostic Assessment This should take about 5 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

Q1. Perimeter is:



A) The perimeter of a shape is the amount of flat surface enclosed by the shape.

B) The perimeter is the length plus the width

C) Perimeter is the distance around the outside (border) of a shape.

D) Perimeter is the amount of space a 3D shape occupies.

C. is the correct answer.
Common Misconceptions
A. Learners often confuse perimeter with area
B. Learners are making 2 mistakes here, believing that perimeter is all to do with rectangles and then only finding half of it.
D. This shows confusion between perimeter and volume.
https://diagnosticquestions.com

Why do this activity?

The main challenge in this problem is to be able to read it, to understand the information given and to understand what has to be found. Learners will get practice in reading comprehension and in 'visualizing' and 'mathematizing' the problem. With a good teacher learners will be accustomed to having to think for themselves and most learners will not even want a hint. Many learners will have the pleasure of the 'AHA' moment when the light dawns and they understand how to do the problem.

This problem only requires an understanding of the concept of perimeter. Confusion between 'perimeter' and 'area' is very common so the problem has the value that it focuses on the common idea of a fence or boundary. Using the mathematical word perimeter in this connection will help learners to remember the meaning of the word.

Learners who know how to work out the circumference of a circle can go on to solve the two further problems Not-so-square Fence and Belt Around the Earth. The surprising fact in all these problems is that the answer is exactly the same whatever the size of the field or the sphere so you do not need to know that size.

Learning objectives

In doing this activity students will have an opportunity to:

- Reinforce understanding of perimeter.
- Develop problem solving skills and the ability to use and apply knowledge to a real life context.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations;
- **visualize**, develop the skill of interpreting and creating visual images to represent concepts and situations and apply this to interpreting information and **solving problems.**

Suggestions for teaching

You might introduce the problem by saying that it is about a fence around a field, saying that a fence like that is called a boundary fence and asking if anyone knows what the word boundary means and if anyone knows the mathematical word for boundary (i.e. perimeter). You might like to talk about the fence around the school grounds, also a boundary fence although the shape inside is not a square. Then ask the learners to read the problem sentence by sentence, to draw their own diagrams and to try to find the solution. You can use the "One-Two-Four-More" teaching strategy (make learners work individually, then in pairs, then in fours, then finally have a class discussion).

For older learners this can be used as a warm-up problem for giving them the problem Not-so-square Fence for which the key idea is similar.

Key questions

Have you drawn a diagram?
When you make the fence longer and the path wider where does the extra fence go?

Follow up

Not So Square Fence: https://aiminghigh.aimssec.ac.za/years-8-12-not-so-square-fence/

Belt around the Earth: https://aiminghigh.aimssec.ac.za/years-8-12-belt-around-the-earth/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 7**	**Secondary 1 and 2**	**Secondary 3 to 6**