NOT-SO-SQUARE FENCE

A fence is built around a square field. Suppose another fence is built exactly one metre longer so the path between the two fences is the same width all the way round including at the corners of the field where the fence goes along an arc of a circle.

1. How wide would this path be?
2. Would a mouse be able to run along this path?
3. Could a farmer drive his cows along the path between the two fences?
4. If you were the farmer and you had already decided on the width that you wanted to make the path how would you work out how much fencing to buy for the outer fence.

Help

The diagram shows that the outer fence is curved at the corners to make the path the same width everywhere. What sort of curve is this? Copy the diagram and mark the 'extra' fence at the corners. What shape do you get if you put the four extra curved bits of fence together? The length of this curve has a different name but it is still the distance all the way round the edge (the perimeter).

Next

See 'Belt around the Earth' problem https://aiminghigh.aimssec.ac.za/years-8-12-belt-around-the-earth/ on the AIMING HIGHER Teacher Network.

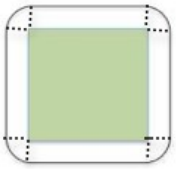
To solve this problem you also need the circumference of a circle. In this problem there is a belt around the Equator and it is stretched by exactly one metre and lifted up so that it is the same distance away from the Earth everywhere. You have to find this distance.

Then you might generalize the problem and investigate what happens if the field is in the shape of an equilateral triangle. How wide would the path then be?

NOTES FOR TEACHERS

SOLUTION

Note: This is the second of a sequence of three problems: Square Fence; Not-so-square Fence and Belt Around the Earth. Each one leads to the next and doing them in sequence will help learners to solve them all and to develop problem solving skills.



This is a question about perimeter. The key to a quick and easy solution here is to understand that the edges of the fence parallel to the edges of the field will always be the same WHATEVER THE SIZE OF THE FIELD and that the extra fence is only needed at the corners.

Method 1 If the width of the path is w, and it is the same all the way around, and you think about the extra fence at each corner, the fence must be curved at the corners around a quarter circle. This means that, for any sized field, the extra fence has length 4 times the length of a quarter circle, which is the length of the circumference of a full circle, that is $2\pi w$ metres. The extra fence has length one metre so the width of the path must be 1 metre divided 2π, that is 0.159 metres or 15.9 centimetres (correct to 3 significant figures). This path is a little wider than the path for the Square Fence problem, and it is wide enough for a family of mice to run along, but not nearly wide enough for cows.

If the farmer wants the width of the path to be say x metres then he must buy sufficient extra fencing to go round the circumference of a circle radius x, that is $2\pi x$ metres of extra fencing (plus a little more for fixing it in place).

Method 2

Expressed algebraically this is

$2\pi w = 1$ metre so $w = 1/2\pi$ metre,

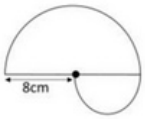
that is 0.159 metres or 15.9 centimetres (correct to 3 significant figures).

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

Two semi-circles are placed next to each other in the design below. What is the total perimeter of the shape?



A $\frac{16\pi}{2}+\frac{8\pi}{2}+8=45.70\ cm$

B $16\pi\times\frac{3}{4}=37.70\ cm$

C $16\pi+8\pi+8=83.40\ cm$

D $\frac{8\pi}{2}+\frac{4\pi}{2}+8=26.85\ cm$

5.If the concept is needed for the lesson the same day, explain the right answer or give a remedial task.
A. is the correct answer.
Common Misconceptions
B. Learners giving this answer probably think that this perimeter is three quarters of the circumference of the bigger circle.
C. Learners giving this answer are calculating complete circumferences rather than lengths of semicircles.
D. Learners giving this answer are working with a formula πr for circumference instead of 2πr.

https://diagnosticquestions.com

Why do this activity?

This problem is about perimeter and in particular it involves using the formula for the circumference of a circle. Like the simpler 'Square Fence' problem, learners need to read and interpret the problem and to "visualize" and "mathematize" it by drawing their own diagrams and thinking mathematically.

With a good teacher, learners will be accustomed to having to think for themselves, and most learners will not even want a hint, but it will help learners to do the Square Fence problem first. Many learners will have the pleasure of the 'AHA' moment when the light dawns and they understand that they must solve the problem by finding the radius of the quarter circles at the corners.

Learning objectives

In doing this activity students will have an opportunity to:

- reinforce understanding of perimeter and circumference.
- practise problem solving and development of reading comprehension and visualization.

Generic competences

In doing this activity students will have an opportunity to:

- work in pairs or small groups to develop **communication and team-working skills**.
- develop the **skill of visualization**, interpreting and creating visual images to represent concepts and situations and applying this to **solve problems in a practical context**.

Suggestions for teaching

You might introduce the problem by saying that it is about a fence around a field, saying that a fence like that is called a boundary fence and asking if anyone knows what the word boundary means and if anyone knows the mathematical word for boundary (i.e. perimeter). You might like to talk about the fence around the school grounds, also a boundary fence. Then ask the learners to read the problem sentence by sentence, to draw their own diagrams and to try to find the solution. You might at some stage ask a learner to draw their diagram on the board and then ask what shape the fence in this question will be. To optimize development of competences you can use the 'One-Two-Four-More' teaching strategy (make learners work individually, then in pairs, then in fours, then finally have a class discussion).

For older learners this can be used as a warm-up problem for the the 'Belt Around The Earth' problem which also involves the circumference of a circle.

Key questions

1. Have you drawn a diagram?
2. When you make the fence longer and the path wider, where does the extra fence go?
3. What shape is the fence in this question?
4. Where is the fence straight? Where is the fence curved?

5. If the path is always the same width what sort of curve is it at the corners?

Follow up

The Circle and The Square https://aiminghigh.aimssec.ac.za/years-6-8-the-circle-and-the-square/
Wholesome Rectangles https://aiminghigh.aimssec.ac.za/years-5-8-wholesome-rectangles/
https://aiminghigh.aimssec.ac.za/years-8-12-belt-around-the-earth/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa.
The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **not** included in the school curriculum for Grade 12 SA.

	Lower Primary Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**