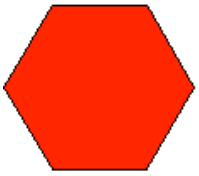
HEXAGON MUTATIONS



Draw chords to cut a regular hexagon

Primary:

(a) into two pieces which, when put together, make a parallelogram;

(b) into three pieces which, when put together, make a rhombus;

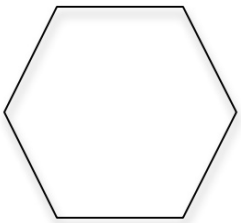
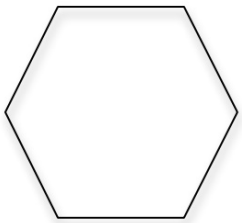
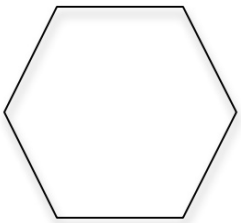
(c) into four pieces which, when put together, make two equilateral triangles.

(d) What angles can you find in each of your shapes?

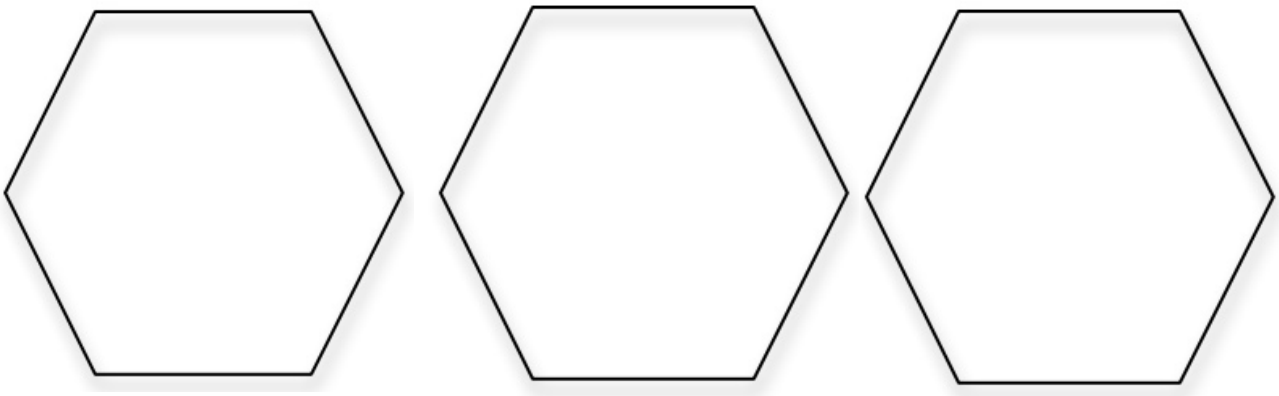
Secondary--- all the above and also:

(e) If the edges of the hexagon are all 2 units what other lengths can you find in each of your shapes?

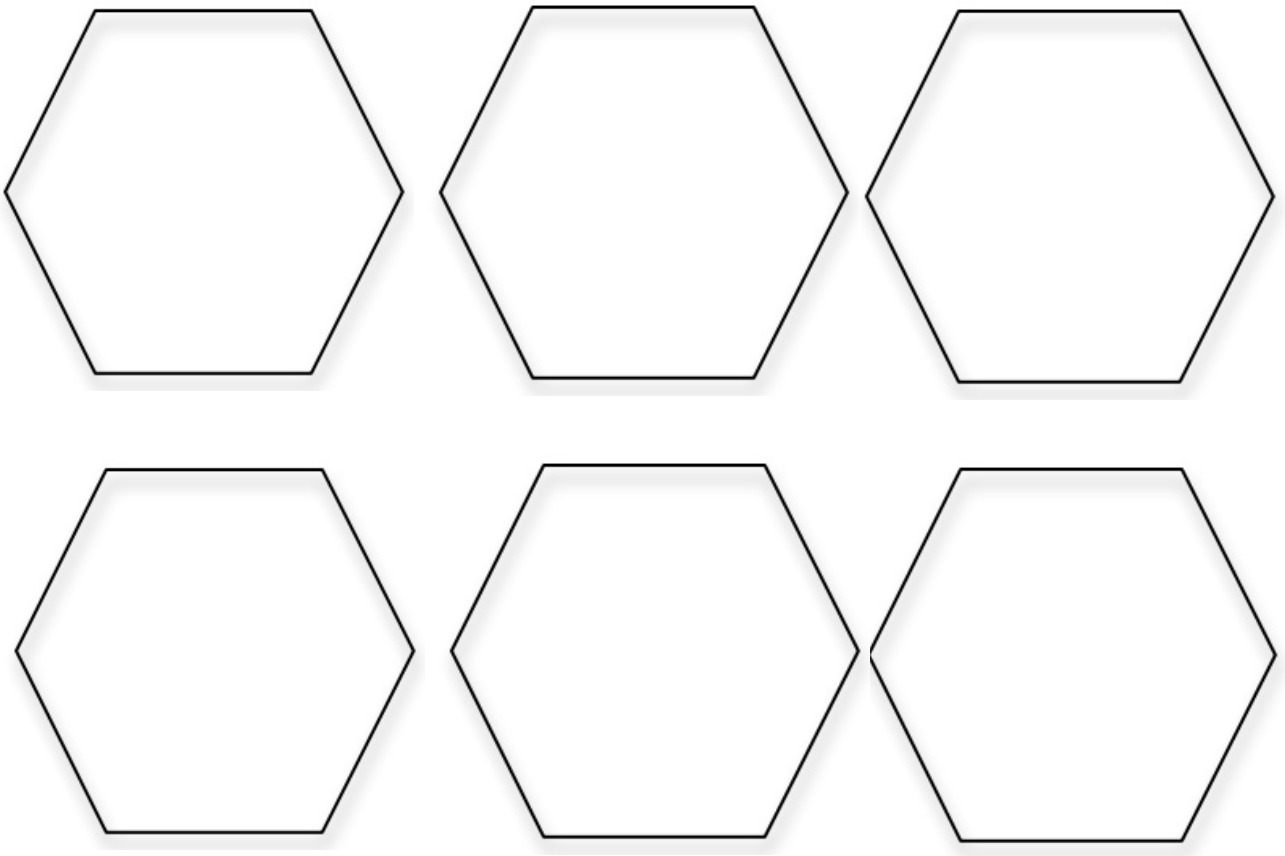
(f) What areas can you find?

Draw chords	**a)** 	**b)** 	**c)** 
Draw new shape	Parallelogram	Rhombus	Two equilateral triangles
Area of new shape			

HELP Cut out these hexagons. Cut them up and use the pieces to make the required shapes.



Here are some more hexagons to experiment with.



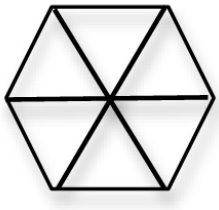
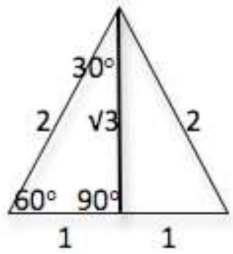
NEXT

The diagrams below show four types of tile, each of which is made up of one or more equilateral triangles. For how many of these types of tile can we place three identical copies of the tile together, without gaps or overlaps, to make an equilateral triangle?

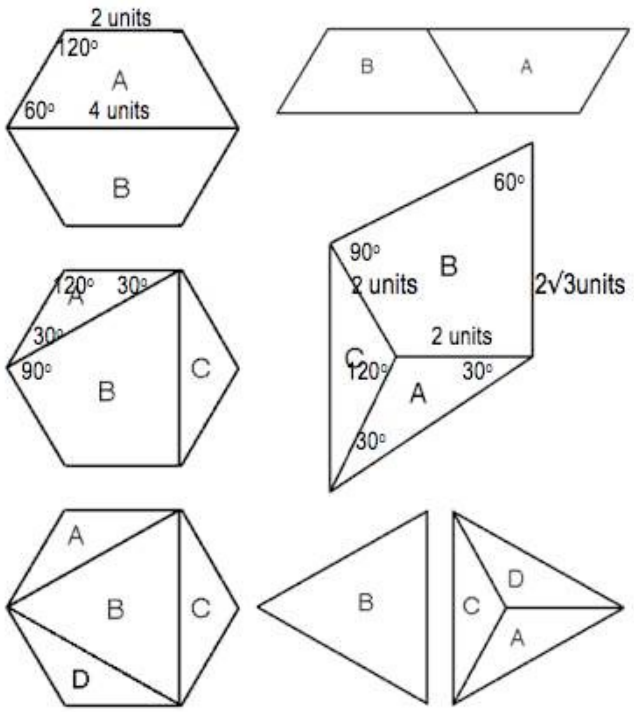
A 0 B 1 C 2 D 4

NOTES FOR TEACHERS

SOLUTION

These facts are useful in solving this problem. 	It is important for all learners to understand that a regular hexagon is made up of 6 equilateral triangles. They should be able to link this to the honeycomb pattern and they should have the opportunity to explore both how regular hexagons can be made from 6 equilateral triangles and how hexagons tessellate the plane. Isometric (triangular) and hexagonal grids are useful. http://www.mathsphere.co.uk/resources/MathSphereFreeGraphPaper.htm	
	By the age of 14 learners should be able to visualise and quickly draw the diagram on the left. They should discover the properties by making an accurate drawing of an equilateral triangle with an axis of symmetry, measuring or reasoning the sizes of the angles and calculating the height using Pythagoras Theorem.	When learners start to work on Trigonometry for a right angles triangle they should be able to write down the sine, cosine and tangent of 30° and 60° without hestation.

SOLUTION



(a) The angles of the parallelogram are 60^o and 120^o.

Length of parallelogram 6 units
Height of parallelogram $\sqrt{3}$ units
The parallelogram has area $6\sqrt{3}$ sq. units

(b) The angles of the rhombus are 60^o and 120^o. The edges of the rhombus are $2\sqrt{3}$ units.
Triangles A and C have height 1 unit and area $\sqrt{3}$ sq. units. Kite B as area $4\sqrt{3}$ sq. units.
The rhombus has area $6\sqrt{3}$ sq. units.

(c) Triangles A, C and D are isosceles with angles of 120^o, 30^o and 30^o;
edges of 2 units, 2 units and $2\sqrt{3}$ units;
height 1 unit and area $\sqrt{3}$ sq. units.

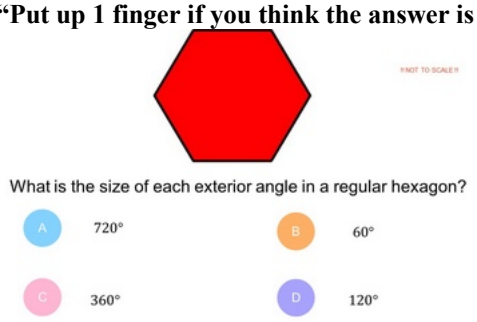
Triangles A, C and D fit together to make an equilateral triangle of area $3\sqrt{3}$ sq. units

Triangle B is equilateral with edges $2\sqrt{3}$ units and area $3\sqrt{3}$ sq. units


These are the two solutions to the question in the NEXT section.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".



2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The correct answer is B
A. The sum of the interior angles is 720°
C. The sum of the interior angles is 360°
D. The interior angles are 120°
https://diagnosticquestions.com

Why do this activity?

This learning activity gives all learners from grades 5 to 12 the opportunity to practice mathematical thinking and visualization. The geometrical ideas are fundamental and learners can discover these ideas for themselves by using triangular and hexagonal grids and cutting out shapes or by accurate drawing and calculations. Confident knowledge of the 30°, 60°, 90° triangle will save learners a lot of time in tests.

Learning objectives

In doing this activity students will have an opportunity to:

- engage in problem solving by using their knowledge of angles to work out how shapes fit together;
- develop visualization skills.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images;
- **work in a team**;
- **present information** and ideas to others.

Suggestions for teaching

WARM-UP OR LESSON STARTER

For older learners this could be a warm-up task or lesson starter that is ready on the board so learners get busy as soon as they sit down in class. It provides a challenge and an opportunity for revision that need not take up much lesson time, especially for upper secondary school students.

Younger learners can be given a grid of hexagons so that they can cut out the hexagons and rearrange the pieces.

REVIEW AND REPORT BACK
You could get learners to work in groups (4 is an ideal number).
Groups could make posters to show their solutions. These can be put on the classroom wall.
When you want to wind up the lesson, and a group has a good solution, then ask them to present their solution to the class. Any mistakes must be treated as learning opportunities. Everyone should help the class to perfect the solution.

Key questions

- What do we know about HEXAGONS?
- What does REGULAR mean?
- How many equilateral triangles fit together to make a regular hexagon?
- What are the angles? How do you know?
- If the edges of the hexagon are 2 units what are the lengths of the chords? How do you know?

Follow up

Tessellating Quadrilaterals https://aiminghigh.aimssec.ac.za/years-7-12-tessellating-quadrilaterals/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6