FLAGPOLES

Two flagpoles are 30 metres apart. One has height 10 m and the other has height 15 m.

Two tight ropes connect the top of each pole to the foot of the other. Draw a diagram.

How high do the two ropes intersect above the ground?

What if the poles were a different distance apart?

Help

You should draw a diagram.

Draw a horizontal line for the ground and draw 2 vertical flagpoles.

Draw 2 straight lines for the ropes.

Mark in the known lengths and distances given in the question.

Mark in your diagram the point on the ground directly underneath the intersection of the two ropes.

Then calculate the distances of this point from the foot of each flagpole.

Extension

'What if the flagpoles were a different distance apart?'

Try different distances. Each member of the group (or class) should choose their own distance and calculate the height of the intersection of the ropes.

Compare answers. Make a conjecture. Try to prove your conjecture.

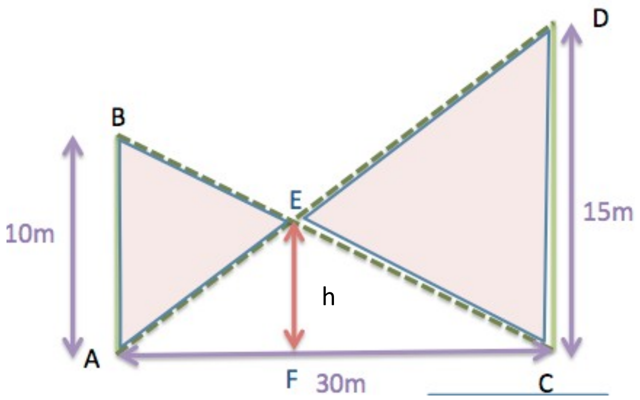
NOTES FOR TEACHERS

SOLUTION

Method 1 Similar triangles

ΔCED is similar to ΔAEB with ratio 15 : 10 = 3 : 2, that is scale factor 1.5

If shapes are similar then they can be positioned to show the centre of enlargement (in this case point E) with lines through the centre joining each point to its image. This is not needed for solving the Flagpoles problem but making connections helps learners to understand the concepts.



CF: FA is the ratio of the heights of ΔCED and ΔAEB so CF = 18 m and FA = 12m.

ΔAEF is similar to ΔADC
so $h/15 = EF/DC = AF/AC = 12/30 = 0.4$

Thus the height h = 6 m.

If the poles were distance d apart then

CF : FA = 3 : 2 gives CF = 0.6d, FA=0.4d
and AF/AC = 0.4 as above.

Thus the height of E does not depend on how far the poles are apart.

Method 2 Trigonometry

$\angle EAC = \tan^{-1} \frac{1}{2} = 26.6^{o}$ which gives $h = 12\tan 26.6^{o} = 6$ metres.

If the poles were distance d apart then $\tan\angle EAC = 15/d$ and, by similar triangles AF = 0.4d so the height $h = AF \tan\angle EAC = (0.4d)(15/d) = 6$ metres.

Other trig. methods that do not make direct reference to similar triangles use similarity intrinsically anyway and are more complicated.

Method 3 Analytical geometry Take A as the origin and AC and AB as x and y axes

AD has equation $y = \frac{1}{2} x$ and BC has equation $y = - x/3 + 10$. These two lines intersect at (12 ; 6).

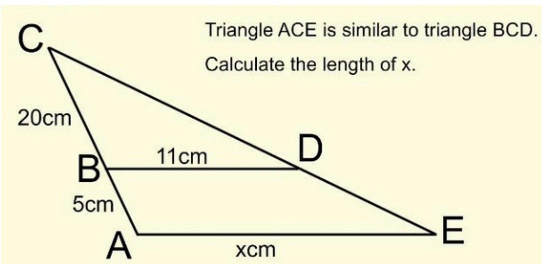
Hence the height EF = 6 metres.

This method does not depend on the distance AC, say d metres.

The lines are $y = 15x/d$ and $y = -10x/d + 10$. These lines intersect at $x = 2d/5$, $y = (15/d)(2d/5) = 6$.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.



A: 13.75 B:16 C: 44 D: 8.8

The correct answer is A

Misconceptions

B. Maybe these students used 11 + 5 = 16.

C. One student explained his answer this way:
The scale factor is 4 worked out by doing 20 divided by 5 which is 4. Then 11 times 4 gives x.

D. Maybe these students used 11 × 4 / 5 = 8.8

https://diagnosticquestions.com

Why do this activity?

This activity can be solved in different ways using the mathematics taught in the school curriculum.
(In South Africa: Trigonometry in Grade 10 Term 1, Similarity in Grade 10 Term 2 and Analytic Geometry in Grade 11 Term 1).
So as to help learners to develop visualisation skills, no diagram is given so that learners get practice in reading and interpreting the information given in the question and drawing their own diagrams. To succeed in examinations learners need plenty of practice in reading and interpreting the information given questions.

Most importantly, if learners are encouraged to think for themselves, they will experience the satisfaction of success in solving problems and become more confident and more motivated.

Learning objectives

In doing this activity students will have an opportunity to:

- develop a deeper understanding of the concept of similar triangles and scale factors through seeing connections with trigonometry and analytic geometry;
- gain practice in drawing diagrams to represent given information.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations and proofs;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations.
- interpret and **solve problems** in a variety of situations;
- **communicate** in writing, speaking and listening
    - o exchange ideas, criticise, and present information and ideas to others
    - o analyze, reason and record ideas effectively.

Suggestions for teaching

LET LEARNERS INVENT THEIR OWN METHODS
There are at least 3 methods here. Apart from the 3 given you can use areas. Do not 'steer' learners to your chosen method. If different learners use different correct methods get them to explain their method to the class in the last phase of the lesson.

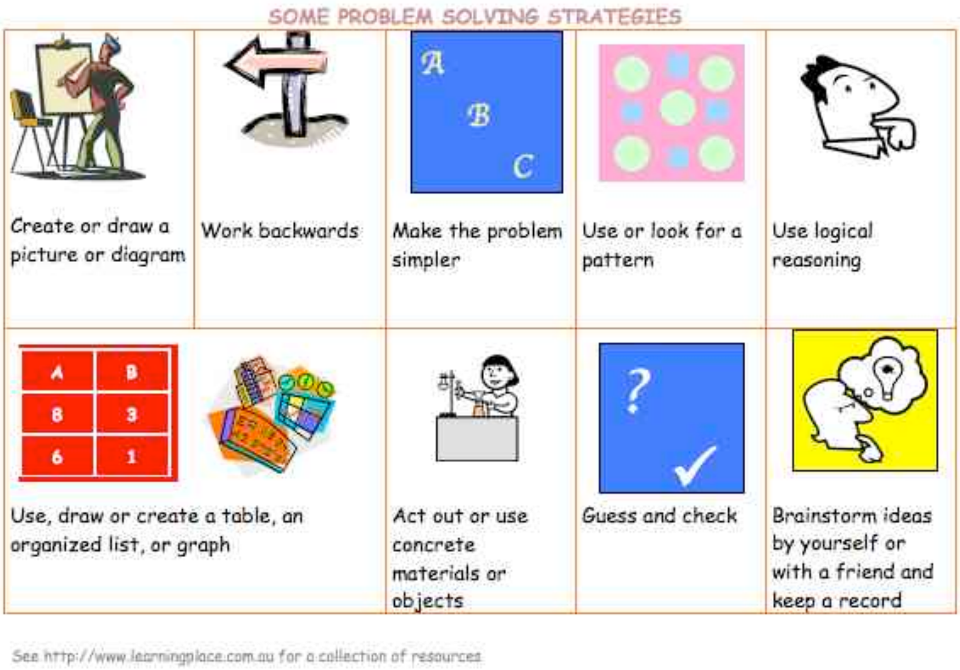
ENCOURAGE LEARNERS TO READ THE PROBLEM AND WORK OUT FOR THEMSELVES WHAT THEY NEED TO DO, PERHAPS IN PAIRS.
Very often teachers say that learners can do the mathematics in class but do badly in tests. One reason for this could be that learners find it difficult to read the question and work out for themselves what they have to do. **Teachers often give too much help** by reading the problem and explaining what the learners must do, perhaps using code switching. You might explain to the learners that they need to get used to reading questions for themselves. Give them this list of how to read the problem and what to ask themselves.

1. Read it slowly. Read it bit by bit
2. "Do I understand all the words?"
3. If not look them up, ask a friend or ask the teacher.
4. "What information am I given?" Make a note of it.
5. Draw a diagram or make a chart of the given information.
6. Sometimes put the given information into an equation.
7. "What am I supposed to find out?"
8. "Have I seen a problem like this before? What did I use to solve that one?"
9. "Do I know any maths that could help?"

PROBLEM SOLVING STRATEGIES

You might get your learners to discuss these strategies and what they involve.

You and your learners could make a poster for your classroom wall as a reminder of the different problem solving strategies. You don't use all of them for every problem but all of them are useful. Your learners might suggest other strategies.



At the end of lessons you might ask them to tell you which strategies they used and how they used them.

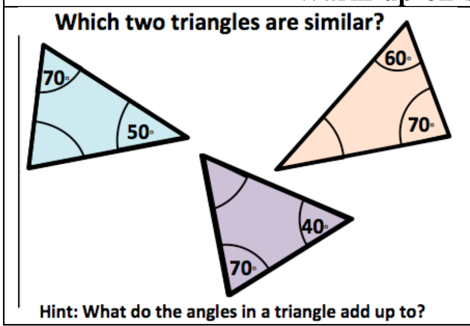
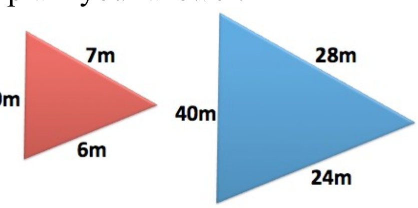
Then you might like to add your comment to the discussion that follows this problem on the AIMING HIGH Teachers' Network site to share with other teachers how best to help learners to be successful problem solvers.

DRAWING THE DIAGRAM

A diagram is essential for this problem. Ask all the learners to read the problem and draw a diagram then ask some learners to come and draw their diagrams on the board. Get the learners to talk about the features of the diagrams. Which is the best one to use? Have they put all the given information into the diagram?

WHAT TO LOOK FOR

It is good practice every lesson to use a 'Warm Up' task or 'Lesson Starter' that is ready on the board so learners get busy as soon as they sit down. The Diagnostic Quiz given above serves this purpose. Your 'Warm Up' can involve some mathematics that could be applied to the main task of the lesson. If you give the Flagpoles problem to a Grade 10 class you might give a 'Warm Up' on similar triangles. If it is a Grade 11 Class it might be on equations of lines. Tell a Grade 12 Class that there are at least 3 methods and you hope that between them they will find the methods without needing a hint from you.

Warm-up on similar triangles		**Warm-up on straight lines**
	Are these triangles similar? Explain your answer. 	Lines a and b intersect at (5 ; -2). Lines b and c intersect at (-1 ; 6). What is the gradient of line b? **Where does line b intersect the line $y = x$?**

REPORT BACK

In the last phase of the lesson you might ask different learners to present their solutions on the board. Seeing different methods is so beneficial for learners that this activity could take up two lesson periods.

Key questions

- What are you trying to find out?
- What do you notice in the diagram that might help? What lengths are given?
- What properties of the triangles in this diagram can you see?
- Can you calculate any other lengths?

Follow up

More trigonometry in right angled triangles : https://aiminghigh.aimssec.ac.za/year-10-cablecar/
https://aiminghigh.aimssec.ac.za/years-10-11-at-sea/
https://aiminghigh.aimssec.ac.za/years-10-11-cliff-rescue/

More on similar triangles:
https://aiminghigh.aimssec.ac.za/grades-8-to-10-simsets/
https://aiminghigh.aimssec.ac.za/years-9-11-kissing-triangles/
https://aiminghigh.aimssec.ac.za/years-11-12-why-the-same/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6