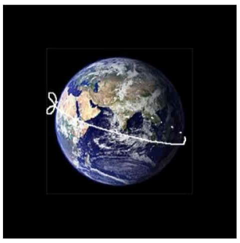
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)

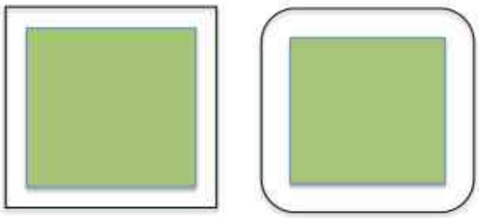
AIMING HIGH

BELT AROUND THE EARTH



A wire belt is tied tightly around the Earth at the equator.
Suppose that another belt is made exactly one metre longer and held around the Earth at the equator so that it is the same distance away from the Earth everywhere.
Would a mouse be able to crawl under the new belt? What about a cat chasing the mouse? How do you know? Show your calculations.
Would the answer be the same for the moon? Why or why not?

Help

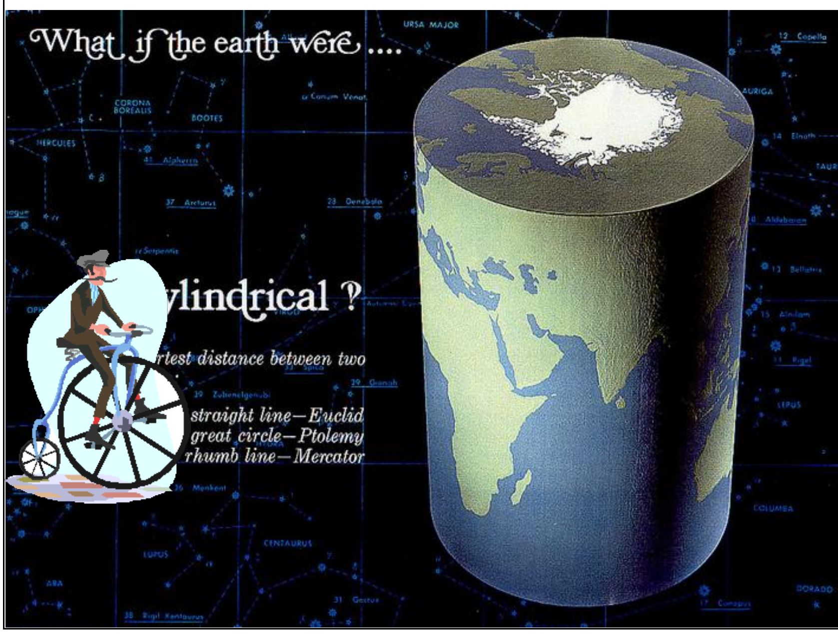


You might find it helpful to try first the very similar problems 'Square Fence' and the 'Not-So-Square Fence' that lead step by step to this 'Belt Around The Earth' problem. The two diagrams show square fields. In each case the outer fence (like the belt) is 1 metre longer than the boundary of the field, and you have to find the width of the path. In the 'Not-So-Square-Fence' problem the fence at each corner is a quarter circle.

Next

You could generalise this problem by investigating fields of different shapes. What is the width of the path if the outer fence is exactly one metre longer than the length of the boundary of the field? Does the size of the field make any difference, or do all fields of a similar shape have the same width of outer path? Can you prove it? This is exactly how mathematicians do research, generalising problems and creating new mathematical results.

See 'Earth Shapes' on the NRICH website http://nrich.maths.org/1363 . This is an imaginative science fiction idea but it provides some further food for thought.



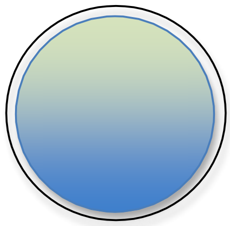
Alternatively another extension problem could be 'Watching the Wheels Go Round and Round' on the NRICH website. http://nrich.maths.org/1039 .

The front wheel on the penny-farthing bicycle has a circumference of 200 centimetres and the back wheel 50 centimetres.

How many times would the wheels turn if the bicycle travels one kilometre? Would the large or the small tyre get more wear and tear on a long journey?

NOTES FOR TEACHERS

SOLUTION



Imagine a slice cut through the equator and model this by taking the land surface to be a sphere and the circumference of the slice to be a circle. It is not a perfect circle but that is a good enough approximation. Then the belt is a circle just one metre longer than the circumference of the earth.

Let the radius of the Earth be given by R metres and the height of the belt above the ground be given by h metres then considering the circumferences of the Earth and the length of the belt in metres:

$2\pi(R + h) = 2\pi R + 1$

So this gives $2\pi h = 1$ metre for all values of R.

$h = 1/2\pi$ metre, that is 0.159 metres or 15.9 centimetres (correct to 3 significant figures).

This gap under the belt is plenty high enough for a mouse to walk underneath with his head held high, even for a small cat with his head down low stalking the mouse.

This is the black footed cat *Felis Nigripes*, the smallest African wild cat. The adult male shoulder height is 25 cm. It lives in dry, open grassland and Karoo semidesert. If the belt around the Earth was on a great circle crossing the Karoo this cat could get under it to catch a mouse, but only by crouching low.

Because the calculation does not use the radius R the answer would be the same for the moon or even for a sphere as small as a soccer ball.

Diagnostic Assessment

This should take about 5–10 minutes.

Write the question on the board, say to the class:

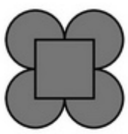
"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.

Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.

Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers. It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.

The diagram shows four equal discs and a square. Each disc touches its two neighbouring discs. Each corner of the square is positioned at the centre of a disc. The side length of the square is $2/\pi$. What is the length of the perimeter of the figure?



A 4 B $\frac{3\pi}{2}$ C 6 D 2π

5.If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

C. is the correct answer :

$4 \times \frac{3}{4}$ circumference $= 4 \times \frac{3}{4} \times \pi \times 2/\pi = 6$

Common Misconceptions

A. The diameter of the discs is the side length of the square and the circumference is 2. The mistake here is finding 4 semicircle lengths.

B. and **D**. Misuse of the formula for circumference.

https://diagnosticquestions.com

Why do this activity?

This problem requires learners to visualize a 3D problem and to think about turning it into a 2D problem because all they need to work on is the circular slice through the earth at the equator. It is an interesting application of finding the circumference of a circle. With a good teacher, learners will be accustomed to having to think for themselves, and most learners will not even want a hint, but it will help learners to do the Not-So-Square Fence problem first.

Learning objectives

In doing this activity students will have an opportunity to:

1. reinforce visualisation in 3D.
2. reinforce knowledge and understanding of the sphere and its properties.
3. develop problem solving skills.

Generic competences

In doing this activity students will have an opportunity to:

- **think flexibly**, be creative and innovative and apply knowledge and skills;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations;
- **develop communication and team working skills**.

Suggestions for teaching

You might use the Not-So-Square Fence problem as a warm-up at the start of a lesson. Then the class could discuss the solution to that easier problem before going on to do the Belt Around The Earth problem.

Ask the learners to read the problem by themselves and to draw their own diagrams. Then lead a class discussion about modeling and about mathematicians having to make approximations in order to model a real situation. Ask what the learners would assume about the shape of the Earth in order to do the question. You could use an orange and cut it in half to demonstrate the key idea in this problem that the equator is a circle.

Then ask the learners to read the problem again sentence by sentence and to draw a diagram with a disc to represent the cross section of the Earth and a concentric circle a little way out for the belt. You can use the "One-Two-Four-More" teaching strategy so make learners work individually to try to find the solution, then in pairs, then in fours, then finally have a class discussion. You might ask a pair of learners who have solved the problem to present their solution to the class. When learners do this always stress that they must explain every step and give reasons for everything they say.

Key questions

1. Have you drawn a diagram?
2. What shape is the 'belt' in this question?
3. What shape is the belt if you make it a bit longer than the equator and always the same height h above the ground?
4. If the radius of the earth is R metres and the belt is h metres out from the Earth what is the radius of the belt?
5. What other information are you given?
6. Would the answer be the same for a belt around the moon?

Follow up

Watching the wheels go round and round http://nrich.maths.org/1039 .
Turning cogs https://aiminghigh.aimssec.ac.za/years-6-10-turning-cogwheels/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6. The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA. For resources for teaching A level mathematics see https://nrich.maths.org/12339				
	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6