## AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES

SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

## LOADING LORRIES



Twenty one gas cylinders are to be loaded onto three lorries.

Seven cylinders are full, seven are halffull and seven are empty.

A full cylinder weighs 50 kg and an empty cylinder weighs 20 kg .

How should they be loaded onto the lorries so that each lorry is carrying the same weight?

## Can you find more than one answer?

## HELP

| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 kg | 20 kg | 20 kg | 20 kg | 20 kg | 20 kg | 20 kg |


| $1 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 kg | 35 kg | 35 kg | 35 kg | 35 kg | 35 kg | 35 kg |


| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 kg | 50 kg | 50 kg | 50 kg | 50 kg | 50 kg | 50 kg |

Cut out the 21 pieces showing the weights of the cylinders and arrange them into 3 sets to load on the 3 lorries so that each set has the same total weight.

## NEXT

Make up a similar problem with a different number of cylinders and different weights or a different number of lorries.

## NOTES FOR TEACHERS

## SOLUTION

## Method 1

Each lorry must take 7 cylinders and in that way the weight of the cylinders alone will be the same.

Seven full and seven half cylinders are equivalent to $101 / 2$ full cylinders so a total of $31 / 2$ cylinders of gas must be loaded onto each lorry. To get $31 / 2$ cylinders each lorry must take an odd number of half cylinders. The only way that the half cylinders can be split between the 3 lorries are $1,1,5$ or $1,3,3$.

This shows how one lorry could be loaded.
Alternatively we can show the answer in a list:

$$
\begin{array}{lllllll}
1 & 1 & 1 & 1 / 2 & 0 & 0 & 0
\end{array}
$$

$$
(1,1,1,1 / 2,0,0,0)
$$

The two ways of loading the lorries are:

| either $(1,1,1,1 / 2,0,0,0)$ | $(1,1,1,1 / 2,0,0,0)$ | $(1,1 / 2,1 / 2,1 / 2,1 / 2,1 / 2,0)$ |  |
| :--- | :--- | :--- | :--- |
| or | $(1,1,1,1 / 2,0,0,0)$ | $(1,1,1 / 2,1 / 2,1 / 2,0,0)$ | $\left(1,1, \frac{1}{2}, 1 / 2,1 / 2,0,0\right)$ |

## Method 2

$50-20=30 \mathrm{~kg}$ of gas so there is 30 kg of gas in a full cylinder and 15 kg of gas in the half-full cylinders.

The half full cylinders must then weigh $20+15=35 \mathrm{~kg}$.
The total weight is $(7 \times 50)+(7 \times 35)+(7 \times 20)=735 \mathrm{~kg}$ so each of the 3 lorries must take 7 cylinders weighing a total of 245 kg .

This tells us that each lorry must have an odd number of half-full cylinders.
The only way that the 7 half cylinders can be split between the 3 lorries are $1,1,5$ or $1,3,3$.

The weights of the cylinders on the lorries must be:
Either
$(50,50,50,35,20,20,20) \quad(50,50,50,35,20,20,20) \quad(50,35,35,35,35,35,20)$
or
$(50,50,50,35,20,20,20) \quad(50,50,35,35,35,20,20)(50,50,35,35,35,20,20)$

## Why do this activity?

The main challenge in the problem is to be able to read and understand the information given, and to understand what has to be found out. Learners will get practice in reading comprehension and in 'mathematizing' information. This is also a real world problem showing the use of maths in common work situations.

## Learning objectives

Learners will practice problem solving involving fractions and simple proportions.

## Generic competences

Learners will get practice:

- in reading comprehension and in 'mathematizing' information:
- in real world problem showing the use of maths in common work situations;
- in teamwork if they work in a group.

DIAGNOSTIC ASSESSMENT This should take about 5-10 minutes.
Write the question on the board, say to the class:
"Put up 1 finger if you think the answer is $A$, 2 fingers for $B, 3$ fingers for $C$ and 4 for $D$ ".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers. Putting thoughts into words may help them to gain better understanding and improve their communication skills.
 and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
3. Ask the class to vote for the right answer by putting up $1,2,3$ or 4 fingers. Notice if there is a change and who gave right and wrong answers.
4. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

A is the correct answer.

## Common Misconceptions

B. you times 6 by 2 because you add 2 more people
D. because 4 ADD $2=6$ and 6 eggs times $6=36$
https://diagnosticquestions.com

## Suggestions for teaching

Ask the learners to read the problem sentence by sentence. Get them to say what they understand from that sentence. What information are they given? What do they have to find out? Discuss the information until everyone understands that they must work out how to load onto 3 lorries with:
21 gas cylinders
7 full weighing 50 kg
7 half-full
7 empty weighing 20 kg
Ask how they could find the weight of a half full cylinder?
Draw the diagram on the board. Explain that the learners should now put the 1 's, $1 / 2$ 's and 0 's onto the lorries so that each lorry has the same number of cylinders and the same weight. You may want to give out cards cut out from page 4 so that learners can try out their ideas by moving the cards around.

| 1 | 1 | 1 | 1 | 1 | 1 | 1 | $1 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ | $1 / 2$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 50 kg | 50 kg | 50 kg | 50 kg | 50 kg | 50 kg | 50 kg | 35 kg | 35 kg | 35 kg | 35 kg | 35 kg | 35 kg | 35 kg | 20 kg | 20 kg | 20 kg | 20 kg | 20 kg | 20 kg | 20 kg |



LORRY 3

## Key questions

- What is the weight of gas in one full cylinder ( $50-20=30 \mathrm{~kg}$ )
- What is the weight of one half-full cylinder $(20+15=35 \mathrm{~kg})$
- What is the total amount of gas? ( $101 / 2$ cylinders 315 kg weight of gas)
- How much gas will go onto each lorry? ( $31 / 2$ cylinders, 105 kg )
- What is the total weight to be loaded ( 735 kg )
- How much weight will go onto each lorry? $(245 \mathrm{~kg})$
- Can you find more than one solution?
- How do you know that you have found all the solutions?


## Follow up

The learners should work in groups to make up a similar problem with a different number of cylinders and different weights or a different number of lorries. Then each group should do the problems made up by all the other groups.

Go to the AIMSSEC AIMING HIGH website for lesson ideas, solutions and curriculum

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