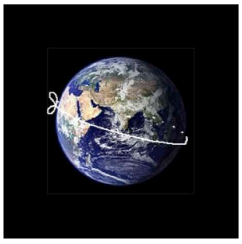
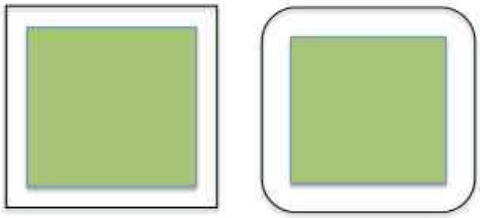
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

BELT AROUND THE EARTH



A wire belt is tied tightly around the Earth at the equator.
Suppose that another belt is made exactly one metre longer and held around the Earth at the equator so that it is the same distance away from the Earth everywhere.
Would a mouse be able to crawl under the new belt? What about a cat chasing the mouse? How do you know? Show your calculations.
Would the answer be the same for the moon? Why or why not?

Help

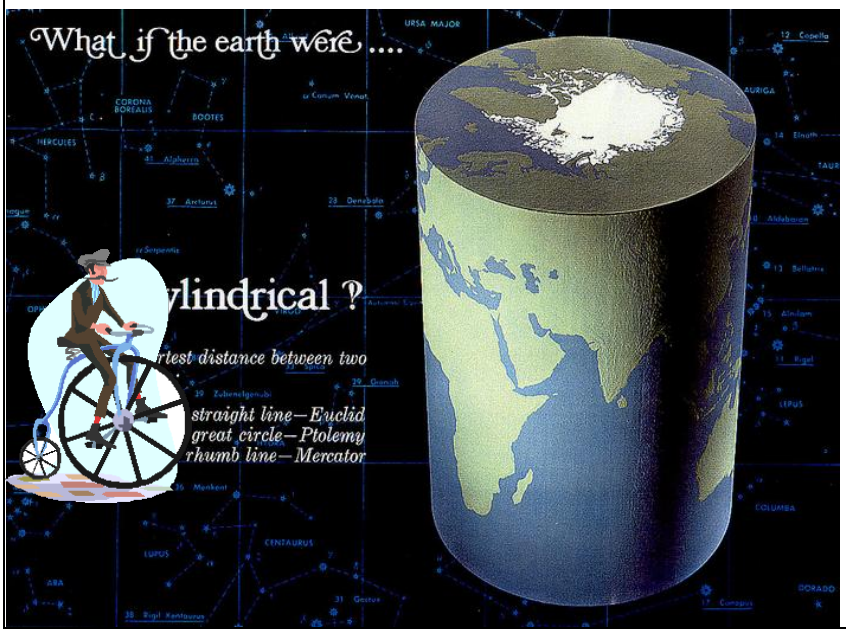


You might find it helpful to try first the very similar problems 'Square Fence' and the 'Not-So-Square Fence' that lead step by step to this 'Belt Around The Earth' problem. The two diagrams show square fields. In each case the outer fence (like the belt) is 1 metre longer than the boundary of the field, and you have to find the width of the path. In the 'Not-So-Square-Fence' problem the fence at each corner is a quarter circle.

Next

You could generalise this problem by investigating fields of different shapes. What is the width of the path if the outer fence is exactly one metre longer than the length of the boundary of the field? Does the size of the field make any difference, or do all fields of a similar shape have the same width of outer path? Can you prove it? This is exactly how mathematicians do research, generalising problems and creating new mathematical results.

See 'Earth Shapes' on the NRICH website http://nrich.maths.org/1363 . This is an imaginative science fiction idea but it provides some further food for thought.



Alternatively another extension problem could be 'Watching the Wheels Go Round and Round' on the NRICH website. http://nrich.maths.org/1039 .

The front wheel on the penny-farthing bicycle has a circumference of 200 centimetres and the back wheel 50 centimetres.

How many times would the wheels turn if the bicycle travels one kilometre? Would the large or the small tyre get more wear and tear on a long journey?