WHERE CAN WE VISIT?

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Choose your number to start and shade it on the grid.
To go to another number you can
either MULTIPLY BY 2 or SUBTRACT 5
So if you start on 12, you can either shade $12 \times 2 = 24$ or $12 - 5 = 7$
Next choose any shaded number and carry on until you can't get to any numbers except numbers already shaded.
Record your results.

1. Choose another number to start. Which numbers can you visit?
2. Can you find a starting number that takes you to all the numbers in the grid?
3. What happens if you use a grid for 1 to 100?
4. Describe at least one pattern that you notice.
5. This puzzle starts with a 5×5 square of numbers and you can either $\times 2$ or -5. Invent a new puzzle by slightly changing something in the rules of this puzzle. Tell us about your puzzle.

Help

You should have no difficulty with doubling or subtracting 5, but may find it more difficult to spot the patterns involved. So use coloured arrows to represent **repeated** subtractions of 5 on some grids (each starting from a different number). It may help you to see the patterns.

Extension

Try other pairs of operations like $\times 3 - 5$ or $\times 4 - 5$ or $\times 5 - 5$ or $\times 6 - 5$.

NOTES FOR TEACHERS

SOLUTION

When any number with the factor of 2 is doubled (multiplied by 2) it gives a units digit of 2, 4, 8, 6, and back to 2. Subtracting 5 takes you to a number with a units digit of 1, 3, 7 or 9.
In a 5 by 5 grid you can visit:

- every number except 21, 23 and multiples of 5 (5, 10, 15, 20 and 25) by starting with any one of the other 18 numbers.
- every number except 23 and multiples of 5 by starting with 21
- every number except 21 and multiples of 5 by starting with 23
- 5,10, 15, 20 and 25 by starting from any one of those numbers but in this case you cannot visit any other numbers.

There is no starting number that takes you to all the numbers in the grid.

In a 10 by 10 grid you can visit:

- every number except 97, 99 and multiples of 5 (numbers ending in 0 or 5) by starting with any one of the other numbers.
- every number except 97 and multiples of 5 by starting with 99
- every number except 99 and multiples of 5 by starting with 97
- all multiples of 5 by starting from any one of those numbers but in this case you cannot visit any other numbers.

There is no starting number that takes you to all the numbers in the grid.

WITH OTHER OPERATIONS

Solution for × 3 and -5:
In a similar way we can find all the numbers except for the multiple of 5's because no numbers can make it to 5, 10, unless we start with a multiple of 5. We can't get 92, 97, 98 because they aren't multiple of 3's and can't get it from -5.

Solution for × 4 and -5:
This is time the numbers that can be reached depend on the numbers started with. When the starting number ends with 1, 4, 6, 9, there are no ways to get any number that ends differently (1×4=4, 4×4=16, 6×4=24, 9×4=36). So we can reach everything ending in 1, 4, 6, 9 except the large numbers 94, 99 because they are not multiples of 4 and we can't get them by -5. To get the numbers 94 and 99 we must start with them.
When the starting number ends with 2, 3, 7, 8, there is no way to get any number that ends differently (2×4=8, 3×4=12, 7×4=28, 8×4=32). So we can reach anything ending in these digits, apart from 93, 97 and 98 which we can get to by starting from 97 or 98,
When the start number ends with 5 or 0, there is no way to get any number that ends in anything other than 0 and 5.

Solution for × 5 and -5:
This reaches every multiple of 5, and the number you start with.
The reason is $5\times x=5x$, which has to end with 0 or 5. No more exceptions after that.

Solution for × 6 and -5:
This is another one that is dependant on the starting number.
If it ends with 1 or 6, you can't get any numbers that don't end with 1 or 6 (1×6=6,6×6=36).
If it ends with 0 or 5, you can't get any numbers that don't end with 0 or 5 (0×6=0,5×6=30). You can't reach 95 and 100 (15×6=90) unless you start from these numbers.
If it ends with 4 or 9: The same thing (4×6=24,9×6=54). There are exceptions 99, 94, 89 (14×6=84).
If the stating number ends with 3 or 8. 3×6=18, 8×6=48 so again you can reach all numbers ending in 3 or 8, but missing 98, 93, 88, 83 as the highest that can be reached is 13×6=78

Diagnostic Assessment

This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

A sequence is given by the rule $u_n + 2u_{n+1} = u_{n+2}$.
The first two terms are 1 and 2.
So $u_3 = 1 + 2 \times 2 = 5$.
Which of these answers shows u_3 u_4 and u_5?

A: 5, 10, 29

B: 5, 12, 24

C: 5, 10, 25

D: 5, 12, 29

The correct answer is **D.**

$u_4 = 2 + 2 \times 5 = 12$

$u_5 = 5 + 2 \times 12 = 29$

https://diagnosticquestions.com

Why do this activity?

This activity gives all learners opportunities to explore fundamental ideas about number theory in a simple context. They are encouraged to explore, conjecture, generalise and justify. There are opportunities for older learners who are familiar with algebraic manipulation or modulo arithmetic to produce rigorous proofs.

Learning objectives

In doing this activity students will have an opportunity to:

- develop a deep conceptual understanding of number;
- review and consolidate knowledge of multiples.

Generic competences

In doing this activity students will have an opportunity to:

1. **think mathematically**, reason logically and give explanations and proofs;
2. **visualize** and develop the skill of interpreting and creating visual images to represent concepts and situations.

Possible approach

You need a set of number cards 1-25, with the multiples of 5 removed or, for older learners, a set of 1-100 cards with multiples of 5 removed.

.Shuffle the cards in front of the class and hand out one to each learner. **Do not tell them that the multiples of 5 are missing!**
Ask each learner to turn to their neighbour and work out how to get from one number to the other and back again, using only these two operations: **×2 and −5**

For example if the two numbers are 22 and 8 the chains could be:
22, 17, 12, 7, 14, 9, 4, **8**, 16, 11, **22**
or **22**, 17, 12, 7, 2, 4, **8**, 16, 11, **22**

Pairs of numbers that are proving difficult to connect could be written on the board and offered as a challenge for the whole class to solve. Everyone **should** be able to arrive at their partner's number but 21 and 23 won't be able to get back!

Finally challenge the class to get to **your** number (which should be a carefully chosen multiple of five). You may wish to offer a prize......

Once the class give up, ask them to explain why it is impossible.

Display the 1-100 grid, choose 42 as your starting number and explain that by using the operations above we are going to try to visit all the numbers on the grid.

Demonstrate how the numbers are crossed out as they are visited.

Ask the students to predict what will happen.

Will they be able to visit every number on the grid at least once?

Hand out this 1-100 grid and allow some time for learners to work in pairs to check their predictions.

Bring the learners together to link their ideas to the findings from the earlier exercise.

What would have happened if they had started on a different number?

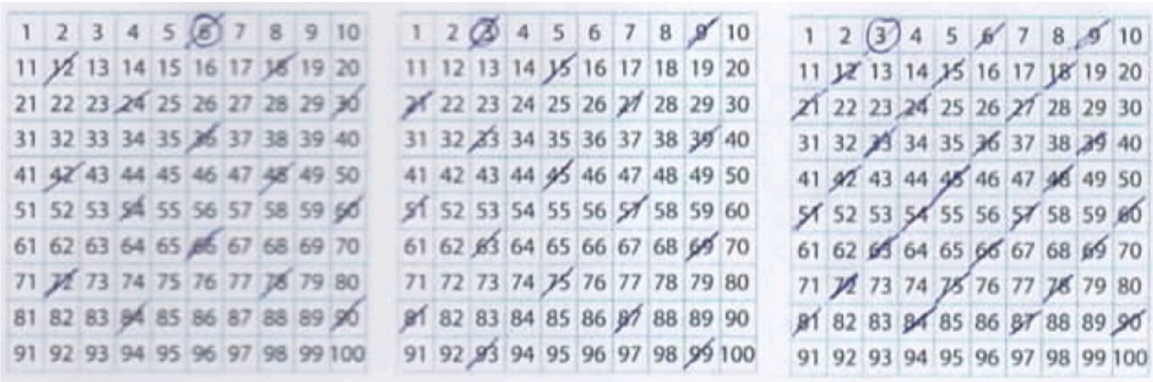
Can they explain their results?

Ask if they think they will get the same sort of results with other pairs of operations.
You may wish to suggest families of pairs of operations for them to explore, for example:
x3 and -5
x4 and -5
x5 and -5...
or
x5 and -2
x5 and -3
x5 and -4... or they can try some families of their own choosing.

Hand out plenty of the 1-100 grids and ask learners to work in pairs or small groups and make a display of their results.

Can they explain their findings and use these to begin to make predictions about other pairs of operations? Encourage them to justify their predictions.

Perhaps see if your learners can identify the pair of operations that produced the patterns in these three grids:



They can choose from either:
×3 and −6,
or ×6 and −3
See if they can spot which is which, and if the starting number makes a difference.

Key questions

- What happens to multiples of 5 when they are doubled?
- What about numbers that are 1 more, 2 more, 3 more and 4 more than a multiple of 5?
- What happens to multiples of 5 when 5 is subtracted from them?
- What about numbers that are 1 more, 2 more, 3 more and 4 more than a multiple of 5?

Follow-up ideas

Beautiful number patterns and deeper understanding: https://aiminghigh.aimssec.ac.za/years-6-9-times-nine/
For younger learners: https://aiminghigh.aimssec.ac.za/years-5-8-multiple-patterns/
Building on what learners know about times tables and leading them towards an understanding of functions: https://aiminghigh.aimssec.ac.za/years-7-9-shifting-times-tables/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**