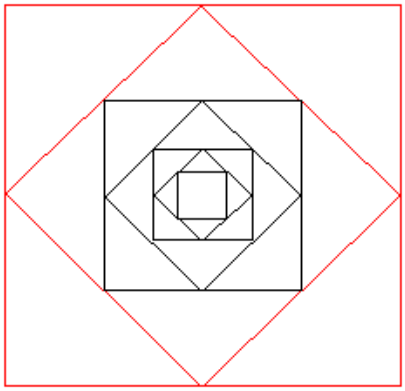
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

SEVEN SQUARES



Seven squares are drawn inside each other.

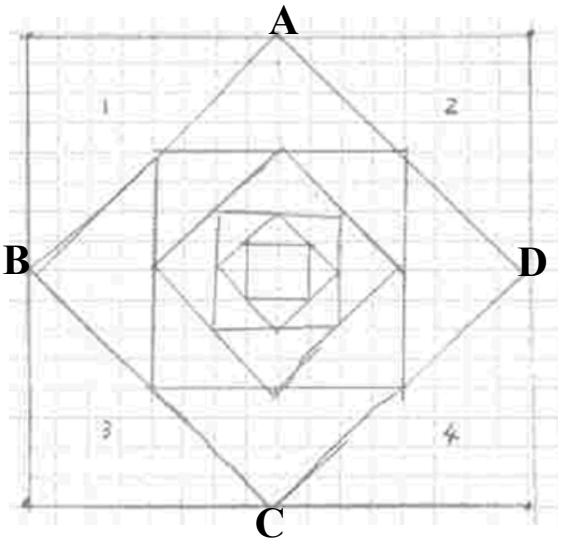
The centre points of each side of the outer square are joined to make a smaller square inside it and so on.

The centre square has the area of 1 (one) square unit.

Draw the diagram. You can download square dotty paper here.

What is the total area of the four outside triangles outlined in red?

SOLUTION



The outer triangles numbered 1, 2, 3 and 4 in the diagram together have the same area as the square ABCD. When you draw a square by joining the midpoints of the edges of a square you get a smaller square scaled down to half the area. (This can be shown by paper folding.)

Starting with the smallest square and working outwards the squares are enlarged by an area scale factor 2 and rotated by 45°. The first, third, fifth and seventh squares are enlargements of each other and have areas 1, 4, 16 and 64.

Working inwards the areas of the squares are halved and working outwards the areas are doubled.

The areas of the 7 squares are 1, 2, 4, 8, 16, 32 and 64 square units.

The total area of the four outside triangles is ½ of 64 square units that is 32 square units.

NOTES FOR TEACHERS

Resources needed.

Scrap paper for folding (one piece for each learner). One centimetre square dotty paper (download here). Sharp pencils and rulers.

Why do this activity?

This activity provides a challenge when you are focusing on drawing lines and shapes accurately, or finding areas either by counting squares or calculation. It calls on learners' understanding of squares and right-angled triangles, and requires working systematically, as well as visualisation.

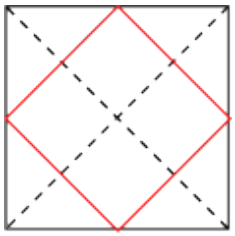
Intended Learning Objectives (Grades 7 or 8)

To draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size

Possible approach

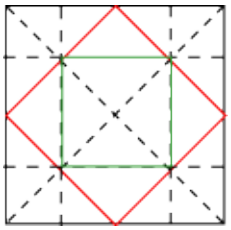
You could introduce this activity using paper folding. Each learner will need a square of paper. (This could be made by folding the end of the sheet diagonally, thus forming a square, and cutting off the excess.)

Take the square of paper and fold it both ways diagonally to find the centre. Fold each corner to the centre and crease it to make the second square (in red).



This shows that the red square has the same area as the total area of the 4 outer squares. Equivalently the red square has half the area of the square of paper you started with.

Next fold each corner to the centre and crease, forming the third square (in green).



Learners could be challenged to continue this process. This introduction will show learners how the squares relate to each other.

Learners should draw the diagram for themselves on square dotty paper. Learners would benefit from working in pairs so that they are able to talk about their ideas with a partner but should do their own drawing.

Learners could decide for themselves the best place to start the drawing and the best size to choose, or the whole group could discuss this together before they begin working individually.

At the end of the lesson learners can discuss both their methods for tackling the drawing and for finding the area of the four triangles. There are several different ways of answering the problem and it can be done without any drawing at all, so there should be plenty to talk about. It might be useful, if no one has suggested it, to cut out the four triangles and put them together as a square.

Key questions

What would be a good way to start drawing- with the smallest square or the largest one?
How big should you draw the centre square to make it easier to draw the others?
Is that a 45^{o} angle?
Now you have done the drawing, how are you going to find the area of the four triangles?

Possible extension

Learners who find this problem straightforward could find the lengths of the edges of all the squares and be asked to work out the linear and area scale factors of the enlargements.

Possible support

Learners could start with a 4×4 inner square and then draw a second square at 45^{o} to this with each side centred on a corner of the first square so that the edges of the second square run diagonally. Then they should draw the third and successive squares in the same way.