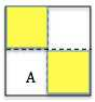
AIMSSEC

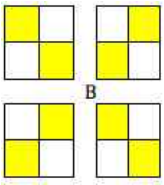
AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

QUARTERS



In how many ways can you half cover one big square using two smaller yellow squares as in diagram A.

What are the similarities and differences between the patterns?



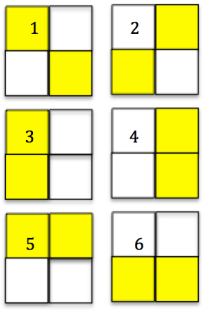
Suppose you half cover four big squares by half covering each square as in diagram B.

Diagram B is unchanged when you rotate it by a quarter turn. How many other 4-square patterns can you find that are unchanged when you rotate the pattern by a quarter turn?

(Thanks to Nicky Roberts for inspiration for this problem)

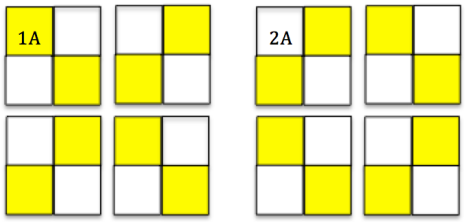
SOLUTION

There are 6 ways to half cover one big square.



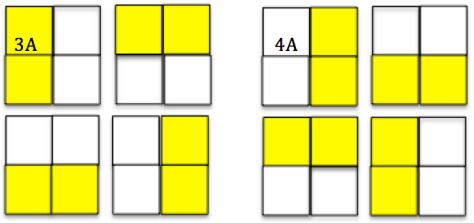
Patterns 1 and 2 are similar. The patterns are unchanged when they are rotated by a half turn (180^o) about the centre so they both have half turn rotational symmetry. They are also symmetrical by reflection in each diagonal. Pattern 1 and pattern 2 differ because they are reflections of each other.

Patterns 3, 4, 5 and 6 are similar. The patterns all have reflection symmetry about a line joining the midpoints of opposite edges but they do not have any rotational symmetry. These patterns differ because they are images of each other after rotations of 90^o, 180^o and 270^o.

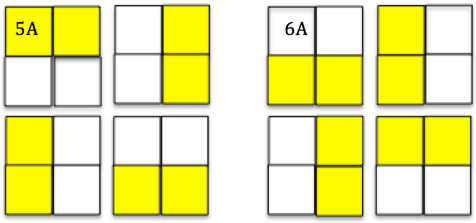


The diagram shows 6 patterns made up of 4 half covered squares. These patterns are all unchanged when rotated by a quarter turn (90^o).

Start with pattern 1 and imagine an axis close to its bottom right hand corner perpendicular to the page. Pattern 1A is made by drawing the images of pattern 1 when it is rotated by a quarter turn (90^o), a half turn (180^o) and a three quarter turn (270^o) about this axis.



Patterns 2A, 3A, 4A, 5A and 6A are drawn similarly and are unchanged when rotated by a quarter turn. These patterns have quarter turn rotational symmetry or rotational symmetry of order 4.



NOTES FOR TEACHERS

Why do this activity?

Learners can do this activity by shading in 2 out of 4 squares on squared paper in different ways. The activity gives them an opportunity to explore different patterns and to look for symmetries and to learn and develop the language of symmetry (reflection, mirror line, line of symmetry, rotation, quarter turn etc.) to describe what they see.

Intended Learning Objectives (Grades 4 to 10)

Learners should recognize, describe and perform translations, reflections and rotations with geometric figures and shapes on squared paper.

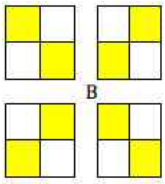
Possible approach

Show the learners this diagram. Ask the learners what fraction of the big square is covered by ONE of the small yellow squares. Then ask what fraction of the big square is covered by TWO of the small yellow squares. Discuss that this uses the idea that 2 quarters is the same as one half.

Give the learners squared paper and ask them to draw squares made up of 4 smaller squares and to find as many different ways as they can to half cover the bigger squares by colouring in two smaller (quarter) squares.

Then get learners to come up to the board and draw the different patterns on the board. When the class have found all 6 patterns ask them to talk with a partner or in a small group about the similarities and differences between the patterns. Then have a class discussion to talk about this and to help the learners to develop their knowledge of the words to use to describe the symmetries that they see.

You might like to stop at this point with primary classes and delay the work on rotational symmetry until secondary school. However this is very visual and practical and easy enough for primary learners to understand. You can print and cut out the larger patterns on pages 3 and 4 and actually rotate them to show the rotational symmetry.



Show the learners this diagram and ask them to describe the symmetries that they see in it. The class should come to understand that this pattern is unchanged when it is rotated by a quarter turn so it has quarter turn rotational symmetry (symmetry of order 4).
There are 4 mirror lines or axes of lines of reflection, a horizontal line, a vertical line and two diagonal lines.

Ask the learners to find other patterns of 4 squares that have rotational symmetry like this.

Discuss the patterns and their symmetries.

Key questions

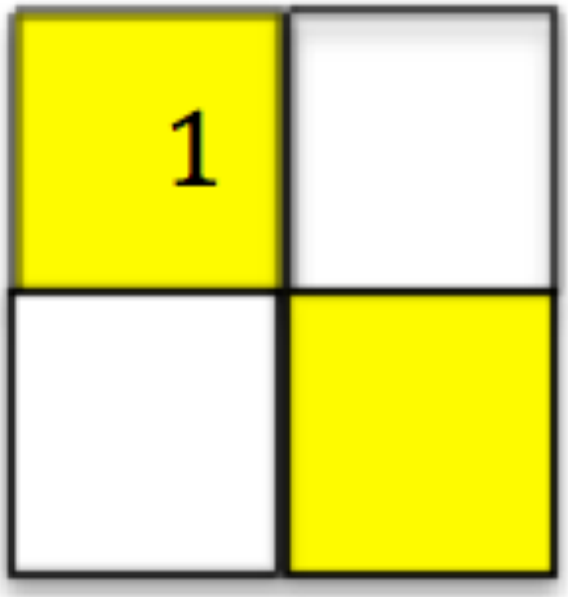
What fraction of the big square is covered by ONE of the small yellow squares?
What fraction of the big square is covered by TWO of the small yellow squares?
Can you show me a mirror line (a line of symmetry).
What happens when you turn Pattern B round by 90^{o}? Does it look the same?
What symmetries can you see?

Possible extension

Learners could find patterns of 4 squares like pattern B but instead of looking for rotational symmetry they could look for reflection symmetry.

Possible support

Print and cut out the big patterns an pages 3 and 4 and then they can be folded in half to show reflection symmetry and rotated to show rotational symmetry.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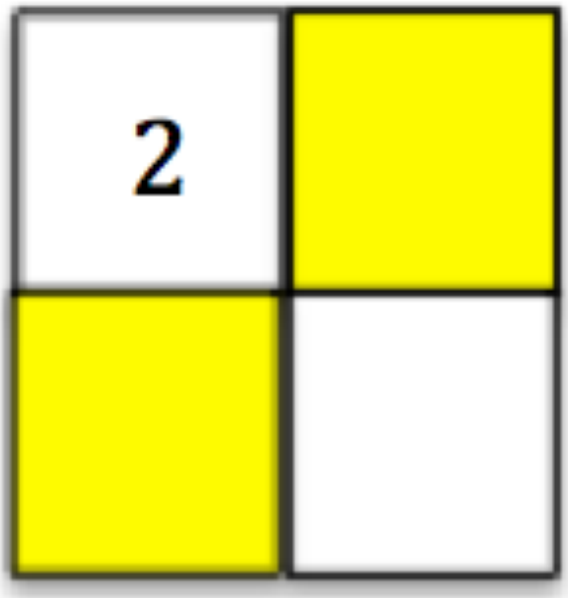
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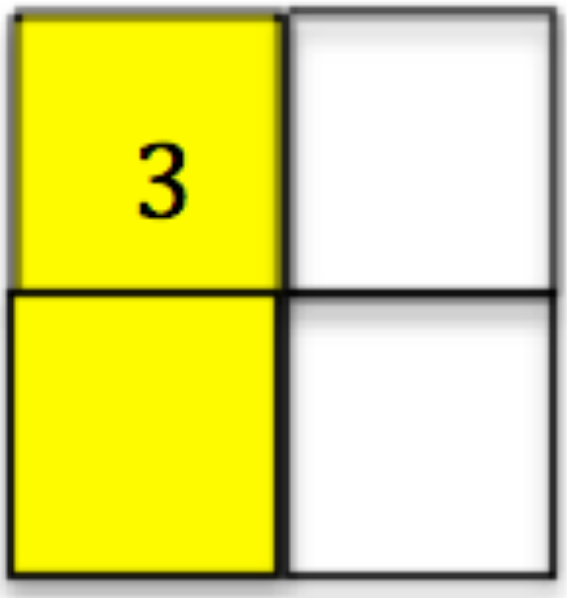
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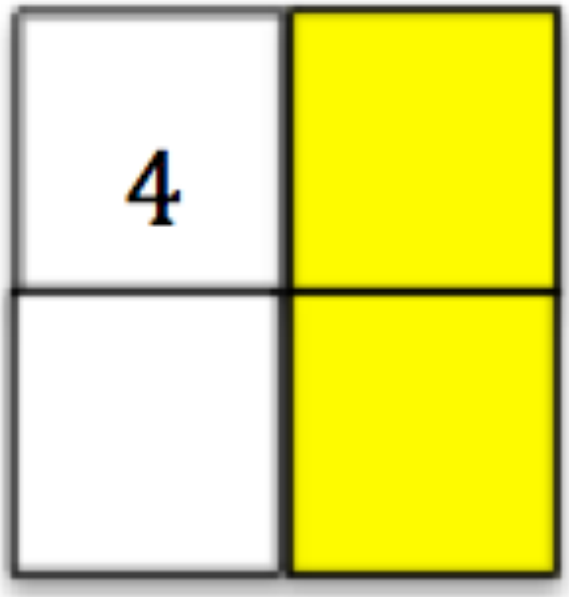
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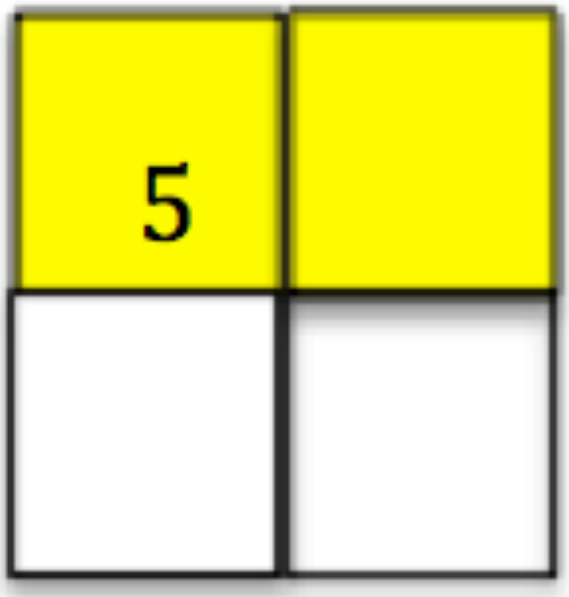
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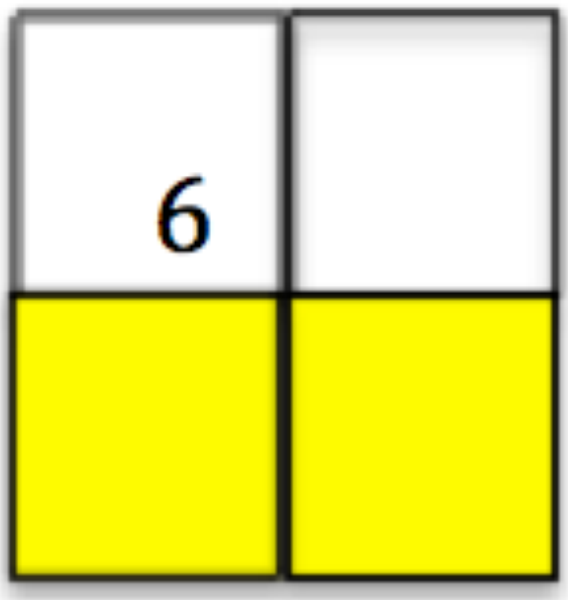
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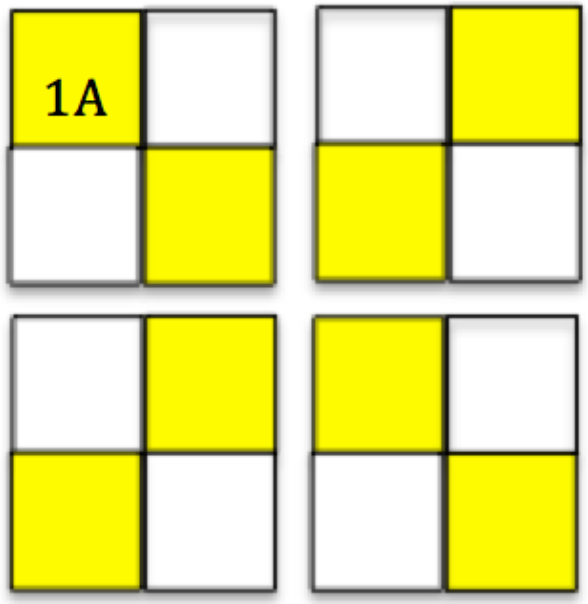
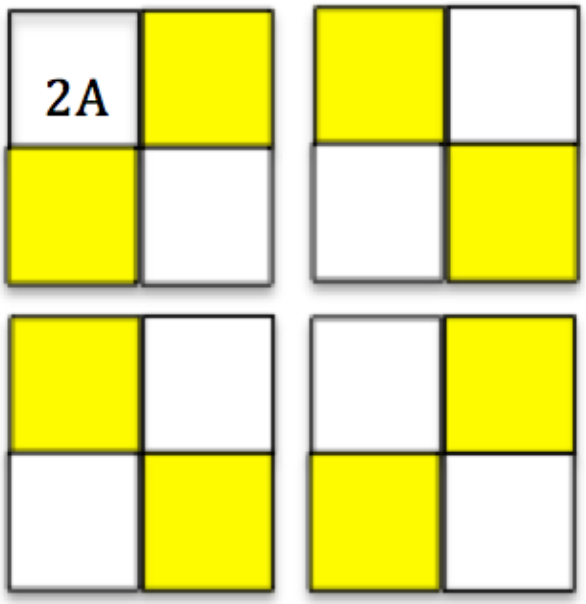

1A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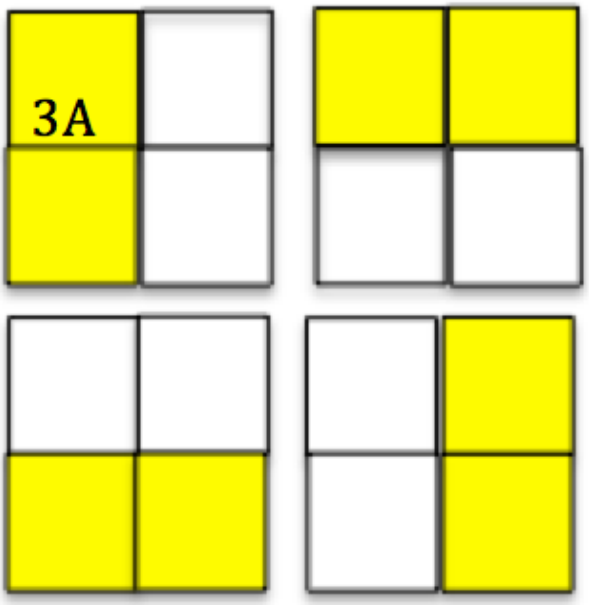
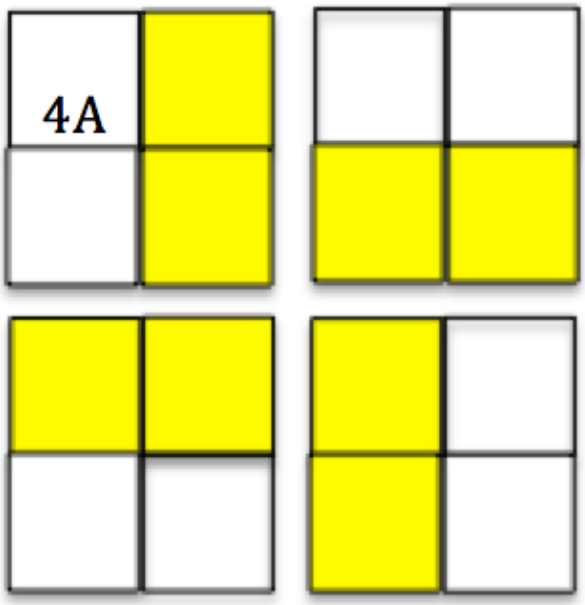

3A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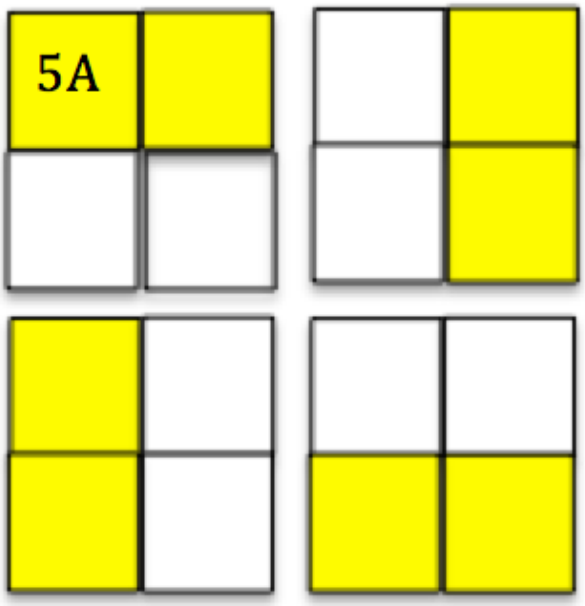
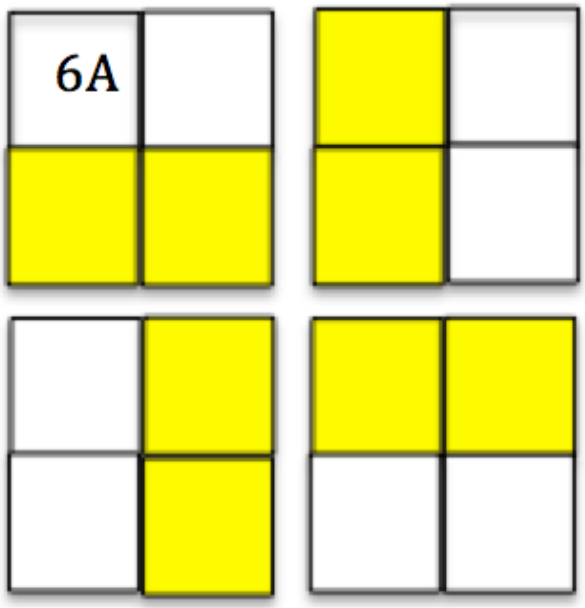

5A


6A