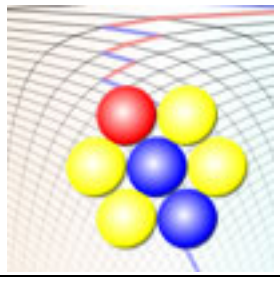
AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

TWOS COMPANY



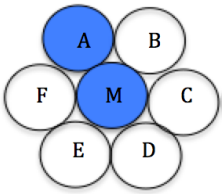
A container holds 4 yellow balls, 2 blue balls and a red ball. The balls are identical in all ways except colour. When you shake the container the balls settle into a hexagonal pattern as shown in the diagram. You win if two blue balls touch.

Would you expect to win less than 50% of the time or more than 50%?

How many outcomes result in a win?

What is the probability of winning?

SOLUTION

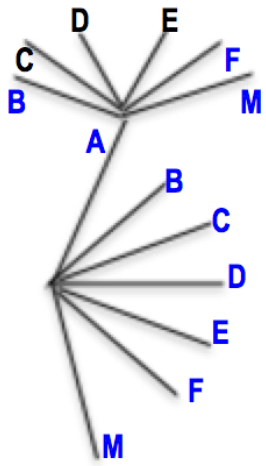


METHOD 1 The important thing here is not to worry where the other balls land but to concentrate on the blue balls.
We need to know how many ways the 2 blue balls can fall and which of these is a win.
If a blue ball falls in the middle then the 2nd ball can land in 6 different places and every one wins.

The places for 2 balls to fall on the edge are:
AB AC AD AE **AF**
BC BD BE BF
CD CE CF
DE DF
EF

Out of these 15 outcomes, the results AB, AF, BC, CD, DE and EF win and the other 9 positions lose.
So there are 21 possible results : 12 win and 9 lose.
The probability of a win is 12/21 = 4/7

METHOD 2 This method uses a tree diagram.



The first blue ball lands in positions A, B, C, D, E, F and M with probability $\frac{1}{7}$.
As shown in the tree diagram, if it lands in position A then the second blue ball lands in positions B, C, D, E, F and M each with probability $\frac{1}{6}$.
Three out of these six positions (B, F and M) are wins.
So the probability of the first ball landing at A and the result being a win is $\frac{1}{7}\times\frac{1}{2}$.

Similarly for the first blue ball landing at B, C, D, E or F.

If the first blue ball lands at M then all positions for the second ball give a win so the probability the first ball landing at M and a win is $\frac{1}{7}$.
So the probability of a win is
$6\left(\frac{1}{7}\times\frac{1}{2}\right)+\frac{1}{7}=\frac{4}{7}$.

As this probability is 0.571 to 3 decimal places, we would expect to win rather more than half the time.

NOTES FOR TEACHERS

Why do this activity?

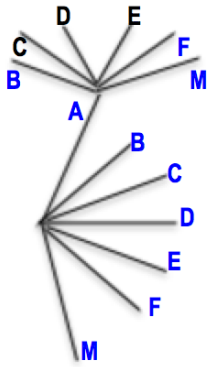
This probability challenge is different from the common type involving dice and cards. Using either a 7-sided spinner or the simulation interactivity on NRICH learners can explore and discuss two types of probability: experimental and theoretical. The computer simulation generates lots of experimental data quickly, freeing time to focus on predictions, analysis and justifications.

Possible approach using a simulation

Simulate the game a few times on the computer or by using spinners which learners can make following the instructions on page 3. You will need to spin the spinner twice to determine where the 2 blue balls fall.

Ask the learners to estimate the probability of winning and allow them some thinking and discussion time in pairs before bringing them together to talk about their initial conjectures. The diagrams on page 3 may help the students to think about the different possible outcomes and the diagrams on page 4 may be useful for demonstration.

Record the learners' conjectures on the board and then run the computer simulation for a hundred shakes. If you are using spinners get each pair to do 5 trials (10 spins) and record the results for the class, for example 150 trials for a class of 30 learners.



Ask the learners to use the extra evidence to refine and justify their conjectures. Run the interactivity for several hundred shakes to test the new hypotheses. Discuss the efficiency of alternative methods of recording the different combinations systematically.

Then ask the learners to work out the theoretical probability of a win and give them some time to do this.

Finally base the plenary class discussion about the probability on the methods that the learners have used.

Possible approach concentrating on the theoretical probability

Give the learners time to work on finding the probability in pairs. If some learners use Method 1 and other learners use Method 2 then ask the learners to explain their methods to the class.

You may want to use this as an example for tree diagrams. In which case you could give the learners a copy of the unfinished tree diagram above and tell them that the blue labels in the tree diagram show the positions of the blue balls. Ask the learners to discuss how the tree diagram could be completed and also ask them to label the probabilities on the branches (but not actually to draw in the remaining branches).

Key questions

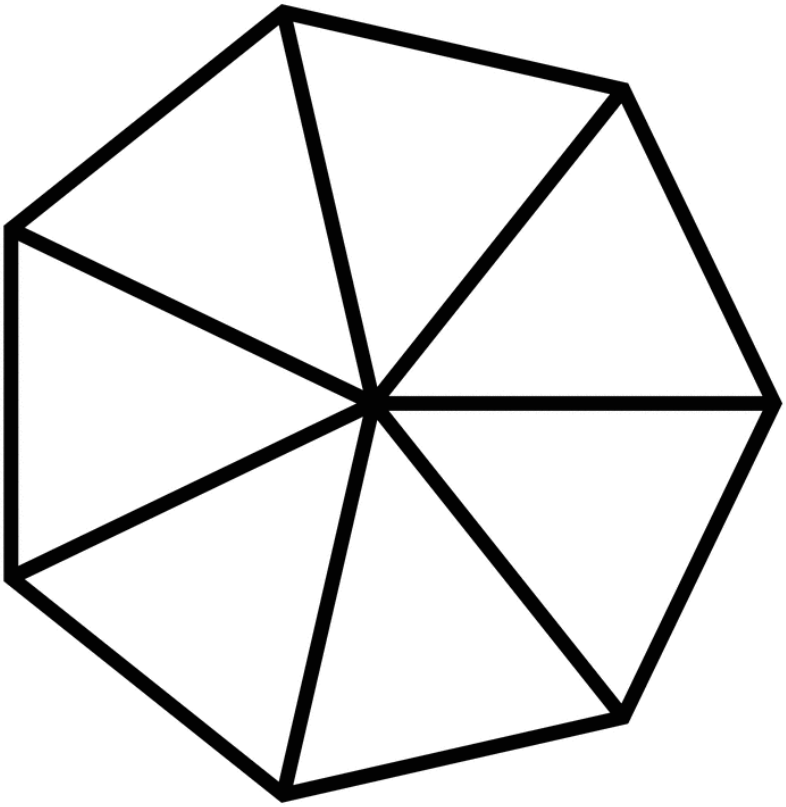
Are there efficient systems for recording the different possible combinations?
What counts as a different outcome?
What are the probabilities on the branches of the tree diagram?

Possible extension

How can we make this game fair?
A follow-up problem could be The Better Bet

Possible support

This problem could be tackled as a follow-up to Cosy Corner



TO MAKE A SPINNER YOU NEED A PIN AND A PAPER CLIP (OPENED OUT)

Cut out the 7-gon and mark A, B, C, D, E, F, M on the sectors.

Pin through the paper clip and 7-gon onto a flat surface so that the paper clip spins freely.