SEVEN CONSECUTIVE NUMBERS

0	1	2	3
4	5	6	
7	8	9	10
11	12	13	
14	15	16	17
18	19	20	

Start with the set of the twenty-one numbers 0 - 20.

Can you arrange these numbers into 7 subsets, each of three numbers, so that the totals of the three numbers in each subset make seven consecutive numbers?

For example, one subset adding up to 27 might be {0, 13, 14}. Another adding up to 28 might be {1, 12, 15}. Find three other numbers that add up to 29 ? These sets are the kind of thing that you need as 27, 28 and 29 are consecutive numbers. Remember that consecutive numbers are numbers which follow each other when you are counting, for example, {24, 25, 26} or {39, 40, 41}.

Hint: Add up all the numbers from 0 to 20. Can you use this sum to decide how big the seven consecutive numbers should be.

There are thousands of number patters that give solutions. What further number patterns can you find? What do you notice about the patterns?

SOLUTION

Understanding the arithmetic mean helps to solve this problem. The sum 0 + 1 + 2 + 3 + … + 20 = 210 As we need 7 ***consecutive*** numbers the middle one will be 210/7 = 30 and the numbers must be 27, 28, 29, 30, 31, 32, 33. These numbers are 13+14, 13+15, 13+16, 13+17, 13+18, 13+19 and 13 + 20.

A	B	C	total
3	10	14	27
1	12	15	28
6	7	16	29
4	9	17	30
0	13	18	31
5	8	19	32
2	11	20	33

A	B	C	total
0	13	14	27
1	12	15	28
2	11	16	29
3	10	17	30
4	9	18	31
5	8	19	32
6	7	20	33

The numbers in columns A & B in both these solutions show all the number bonds for 13 (pairs of numbers that add up to 13). They can be exchanged around in many different ways.

To find all the solutions of this sort we might think of a queue of 7 people. How many ways can the people in the queue re-arrange their order?

The number of arrangements of the pairs of numbers adding up to 13 is like the number of arrangements of the queue. Each one gives a solution to the problem. Think of a tree diagram and how many arrangements there are for the 7 pairs that add up to 13.

We can put with 14 to make 27 any of the 7 pairs 0+13, 1+12, 2+11, 3+10, 4+9, 5+8 or 6+7.

Then there are 6 pairs left to put with 15 to make 28 and this makes 7 x 6 choices so far.

Then there are 5 pairs left to put with 16 to make 29 and this is 7 x 6 x 5 choices so far.

Then there are 4 pairs left to put with 17 to make 30 and this is 7 x 6 x 5 x 4 choices so far.

Then there are 3 pairs left to put with 18 to make 31 and this is 7 x 6 x 5 x 4 x 3 choices so far.

Then there are 2 pairs left to put with 19 to make 32 and this is 7 x 6 x 5 x 4 x 3 x 2 choices.

Then there are 1 pairs left to put with 20 to make 33 and this is 7 x 6 x 5 x 4 x 3 x 2 x 1 choices.

This gives 7! = 7 x 6 x 5 x 4 x 3 x 2 x 1 = 5040 solutions.

There are other solutions too, for example can you complete the solution 4+14+15=33, 1+13+18=32, 3+11+17=31 … ? What further patterns can you find?

NOTES FOR TEACHERS

Diagnostic Assessment
This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The mean of a set of data is 6.
Two more numbers were added.
The mean is still 6.
What can you tell about the numbers that were added?

(A) They must both be even
(B) They add to 12
(C) They must both be less than 6
(D) Can't tell anything

The correct answer is B Students who can make sense of the concept of mean know that the 2 new numbers must also have a mean of 6 and so must add up to 12.

Possible misconceptions.
Getting the correct answer requires understanding of the mean and it is not sufficient just to be able to calculate the mean of a given set of data.
Tests that only check if students can do routine calculations and carry out standard procedures do not test understanding and they are a waste of time.

https://diagnosticquestions.com

Why do this activity?
This learning activity is accessible to learners of different ages and abilities because the only knowledge of mathematics needed is how to add three whole numbers between 0 and 20. There are thousands of different solutions that can be found in different ways, and there is enormous scope for flexible, creative and adaptive thinking and for students to experience for themselves the advantages of working systematically.

Teachers can introduce the activity and challenge the learners to find solutions for themselves. Solutions can be found using an experimental / trial and error approach but some consideration of the structure leads to more efficient solution techniques. There will be no need for learners to feel 'stuck' on this problem: they will always be able to experiment with new combinations.

Learners should be praised for finding new solutions (for example, as new solutions are found each one can be written up on a wall chart). If teachers hold back and just keep asking "can we find any more solutions, how many do you think there are?" and teachers let the learners experiment, then sooner or later a learner may find a way to collect the solutions **systematically.** Alternatively a teacher might help the learners to organize their solutions into tables like the examples given in the solution above, but he / she should only do so after the learners have had plenty of time to experiment with their own methods of organizing and recording solutions.

It may be that learners will surprise the teacher with their ingenuity. In such cases they can share their ideas with the whole class. A good teacher will ensure that his/her students have many similar opportunities to learn from other learners as well as from the teacher.

Learning objectives

In doing this activity students will have an opportunity to:

- practise addition and become more fluent with numbers;
- make sense of the concept of the arithmetic mean and apply it in problem solving;
- develop their own ways of recording their work;
- experience the advantage of working systematically.

Generic competences

In doing this activity students will have an opportunity to:

- engage in independent learning;
- work collaboratively to share ideas and learn with and from other learners;
- have empathy with others, listen to different points of view;
- be creative and innovative - to apply knowledge and skill
- communicate in writing and speaking
    - communicate, exchange ideas, criticise, and present information and ideas to others
    - analyze, reason and record ideas effectively.

Suggestions for teaching

The first step in any problem solving lesson should always be to make sure that the learners understand the problem and what they should be trying to do so learners could be asked to write down the numbers 0 to 20. One learner could be asked to select the first triple and everyone else to write that down and puts a mark by the numbers used that can't be used again. Then all learners must search for a second triple, whose sum is one more than the sum of the first triple. One such triple is chosen, and everyone writes it down and starts to search for the next… until the task of finding triples whose sums are consecutive is fully understood by the group. They will no doubt get to the point where it is impossible to use 3 of the **remaining** numbers to make the next consecutive number. At that point the learners should understand what they are trying to construct and they can work alone or in pairs to find solutions.

The teacher should observe how the learners are working and help and encourage those who have difficulties. At some stage the teacher might decide to ask the whole class to listen to students who describe what difficulties have occured and how they are dealing with them. The teacher can then ask the students to share any observations, or inspirations they have had and also check that the points in the key questions have been covered in the learners' comments.

The challenge is to find a system for listing the many solutions. Learners will be able to discover solutions by trial and error but **can they find a system**? A wise teacher holds back and lets the learners find the system for themselves. If not the class might try the same problem in 12 months time.

The teacher should **not** give too much guidance. The answers are not an end in themselves – the process of finding answers is the important aspect of this lesson. This problem can go on for weeks with a chart on the classroom wall for recording new solutions. Learners who wish to continue to work experimentally could be encouraged to devise a clear way of recording the combinations they are trying. For example, starting with the 20, what are the possibilities for the other two cards, then what about 19? Learners who want to work analytically may choose to use algebra to determine the smallest consecutive number.

Key questions

- Why have you run out of possibilities? Can you change anything to avoid that dead end?
- What are the biggest and smallest sums we could get from a triple?
- Can you work out what the 7 consecutive numbers must add up to?
- What are the 7 consecutive numbers you are trying to make?
- Can you select your triples in a logical/symmetrical way?
- What can you say about the numbers 14 to 20?

Possible extension

- How many different solutions do you think that there might be? Can you work out how many might be possible?
- If the numbers 0 to 20 were changed to different sets of 20 numbers, would solutions still be possible? (try specially designed sets e.g. {10, 11, … 30} or {1000,100, ...1020} or {0, n, 2n, 3n, 4n,... 20n}, or totally random sets).

Possible support

It might be helpful to provide students with cards labelled 0 to 20 to allow them to make their arrangements by placing the cards in 7 rows of 3 cards in a row.

You could also provide calculators so that learners can focus on the structure of the problem, but the practice in addition may be good for those same learners whose are slow with the arithmetic.

Alternative questions include:
Can you find seven **pairs** of numbers which add up to consecutive numbers? Can you find seven sets of three cards which add up to the SAME number?
Students could write such questions for each other, working in pairs to ensure that these are phrased as accurately as possible.

This problem is on the AIMING HIGH Teacher Network http://aiminghigh.aimssec.org/ where you can discuss how to use it in teaching. See also http://nrich.maths.org/2661
You can download a poster at http://nrich.maths.org/6011

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is **beyond** the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	**Upper Primary Age 9 to 11**	**Lower Secondary Age 11 to 14**	**Upper Secondary Age 15+**
South Africa	**Grades R and 1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
USA	**Kindergarten and G1 to 3**	**Grades 4 to 6**	**Grades 7 to 9**	**Grades 10 to 12**
UK	**Reception and Years 1 to 3**	**Years 4 to 6**	**Years 7 to 9**	**Years 10 to 13**
East Africa	**Nursery and Primary 1 to 3**	**Primary 4 to 6**	**Secondary 1 to 3**	**Secondary 4 to 6**