AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE
TEACHER NETWORK

NINES AND TENS

Both 9 and 10 can be made up in 2 different ways by adding pairs of the numbers 1, 2, ... 6, that is

$9 = 3 + 6 = 4 + 5$

$10 = 4 + 6 = 5 + 5$

Explain why it is that when you throw two dice you are more likely to get a score of 9 than of 10.

What about the case of 3 dice?
Is a score of 9 more likely then a score of 10 with 3 dice?

SOLUTION

Method 1

When you throw 2 dice there are 36 possible outcomes.
To get 9 there are 4 outcomes (3,6), (6,3), (4,5) (5,4) so the probability is $\frac{4}{36} = \frac{1}{9}$.
To get 10 there are 3 outcomes (4,6), (6,4), (5,5) so the probability is $\frac{3}{36} = \frac{1}{12}$.

Method 2

Imagine a tree diagram.
To get 9 you must throw 3, 4, 5 or 6 with the first throw (probability $\frac{4}{6} = \frac{2}{3}$.) and then the probability of getting the second number to make 9 is ⅙ so the probability of getting 9 is
$\Pr(9) = \frac{2}{3} \times \frac{1}{6} = \frac{1}{9}$.
To get 10 you must throw 4, 5 or 6 with the first throw (probability $\frac{3}{6} = \frac{1}{2}$.) and then the probability of getting the second number to make 10 is ⅙ so the probability of getting 10 is
$\Pr(10) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.
So as $\frac{1}{9} > \frac{1}{12}$ you are more likely to get a 9 than to get a 10.

FOR 3 DICE

Total of 9

6 arrangements of 1, 2, 6 - {(1, 2, 6), (1, 6, 2), (2, 6, 1), (2, 1, 6), (6, 2, 1), (6, 1, 2)}
6 arrangements of 1, 3, 5
6 arrangements of 2, 3, 4
3 arrangements of 1, 4, 4 - {(1, 4, 4), (4, 1, 4), (4, 4, 1)}
3 arrangements of 2, 2, 5
1 arrangement of 3, 3, 3
25 possible arrangements

Probability of scoring 9 with 3 dice
$= \frac{25}{216} = 0.116$ to 3 decimal places.

Total of 10

6 arrangements of 1, 5, 4
6 arrangements of 1, 3, 6
6 arrangements of 2, 3, 5
3 arrangements of 2, 4, 4
3 arrangements of 2, 2, 6
3 arrangements of 3, 3, 4
27 possible arrangements.

Probability of scoring 10 with 3 dice
$= \frac{27}{216} = 0.125$
so a score of 10 is more likely than a score of 9.

It is not necessary for younger learners to introduce the word 'arrangements' – you could simply call this the number of ways of getting the score.

NOTES FOR TEACHERS

Why do this activity?

This activity gives learners an opportunity for comparing experimental and theoretical probabilities. It also involves thinking systematically about how many different combinations of numbers sum to 9 or sum to 10. Often permutations and combinations are taught formally to older learners who would find the ideas easier if they had met them in simple problem solving situations at an earlier age (without using the technical terms).

Intended learning outcomes

To deepen understanding of probability of 2 and 3 successive events.
To build mathematical thinking and problem solving skills.

Possible approach

Ask learners "If I throw two dice am I more likely to get a sum of 9 or 10?" If you have dice available you can throw them 10 times and record the scores. Ask for a show of hands for those who think 9 more likely and those who think 10 more likely.

For example your results might be: (3,6); (1,1); (6,5); (5,5); (2,1); (3,3); (4,3); (4, 6); (5,3); (2,5) so the experiment gives 9 with a relative frequency of 1 in 10 and it gives 10 with a relative frequency of 2 in 10. As a result of this experiment learners may think the probability of getting a 10 is higher than of getting a 9. To believe that experiments give a true indication of actual probabilities is a common misconception and we have to be careful not to jump to the wrong conclusions. It is important for learners to know this and that it also applies to real life applications.

If you have sufficient dice each pair or small group of learners could throw the dice 10 times and then the class could collect and compare the results for a larger number of experiments.

Then ask the learners how many outcomes are there altogether (how many in the sample space). Then ask them to list the outcomes for 9 and for 10. Give them some thinking time to work out the probabilities then to share their findings in a whole class discussion.

For 3 dice you might ask the class to give you 3 numbers (from 1 to 6) that add up to 9. Write them on the board in a table as shown below. Can anyone suggest 3 different numbers? What about other sets of 3 numbers?... Continue until you have all 6 number triples that add up to 9. Then do the same for 10.

SCORES OF 9		**SCORES OF 10**	
1,2,6	(1,2,6); (1,6,2); (2,1,6); (2,6,1); (6,1,2); (6,2,1)	1,3,6	
1,3,5		1,4,5	
2,3,4		2,2,6	
1,4,4		2,3,5	
2,2,5		2,4,4	
3,3,3		3,3,4	
TOTAL NUMBER OF DIFFERENT WAYS OF GETTING THE SCORE (ARRANGEMENTS)			

Now ask the learner to tell you the different ways you could get a 9 by throwing a die 3 times (or using dice of 3 different colours). Write them in the table.

Then get different pairs or groups of learners each to write down the numbers, in order, that add up to 9 and to 10 : one group to do 1,3,5 another 2,3,4 etc … another to do 1,3,6 etc.

Then record all their answers on the board and count the total number of ways of getting the each score. Finally ask the class how many outcomes altogether (6×6×6=216) and ask them to complete the following:

Probability of 9 = $\frac{?}{216}$ = ? as a decimal, and

Probability of 10 = $\frac{?}{216}$ = ? as a decimal.

Key questions

Can you write those numbers in a different order?
How many ways could the same numbers come up in different orders.
Can you write down the different ways of getting 9 with those two (three) numbers?
Can you write down the different ways of getting 10 with those two (three) numbers?
How many outcomes altogether when you throw 2 dice? Why is it 36?
How many outcomes altogether when you throw 3 dice? Why is it 216?

Possible extension

At Least One https://aiminghigh.aimssec.ac.za/grades-8-to-10-at-least-one/

Possible support

TOTAL SCORE ON TWO DICE						
Red→ Blue↓	**1**	**2**	**3**	**4**	**5**	**6**
1						
2		**4**	**5**			
3		**5**				**9**
4			**7**		**9**	
5		**7**				
6	**7**					

Learners could fill in this table and count the number of outcomes.