NINES AND TENS

Both 9 and 10 can be made up in 2 different ways by adding pairs of the numbers 1, 2, ... 6, that is

$9 = 3 + 6 = 4 + 5$

$10 = 4 + 6 = 5 + 5$

Explain why it is that, when you throw two dice, you are more likely to get a score of 9 than of 10.

What about the case of 3 dice?
Is a score of 9 more likely then a score of 10 with 3 dice?

HELP

TOTAL SCORE ON TWO DICE						
Red→ Blue↓	1	2	3	4	5	6
1						
2		4	5			
3		5				9
4			7		9	
5		7				
6	7					

You could fill in this table and count the number of outcomes.

NEXT

If Busi has 3 children what is the probability that at least one will be a girl?

Imagine tossing a coin three times. What's the probability you will get a head on at least one of the tosses?

What connection is there between the coin tossing experiment and the number of girls in family of brothers and sisters?

At Least One https://aiminghigh.aimssec.ac.za/years-8-10-at-least-one/

NOTES FOR TEACHERS

SOLUTION

Method 1

When you throw 2 dice there are 36 possible outcomes.

To get 9 there are 4 outcomes (3,6), (6,3), (4,5) (5,4) so the probability is $\frac{4}{36} = \frac{1}{9}$.

To get 10 there are 3 outcomes (4,6), (6,4), (5,5) so the probability is $\frac{3}{36} = \frac{1}{12}$.

Method 2

Imagine a tree diagram.

To get 9 you must throw 3, 4, 5 or 6 with the first throw (probability $\frac{4}{6} = \frac{2}{3}$.) and then the probability of getting the second number to make 9 is ⅙ so the probability of getting 9 is $\Pr(9) = \frac{2}{3} \times \frac{1}{6} = \frac{1}{9}$.

To get 10 you must throw 4, 5 or 6 with the first throw (probability $\frac{3}{6} = \frac{1}{2}$.) and then the probability of getting the second number to make 10 is ⅙ so the probability of getting 10 is $\Pr(10) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$.

So as $\frac{1}{9} > \frac{1}{12}$ you are more likely to get a 9 than to get a 10.

FOR 3 DICE

Total of 9

6 arrangements of 1, 2, 6 - {(1, 2, 6), (1, 6, 2), (2, 6, 1), (2, 1, 6), (6, 2, 1), (6, 1, 2)}
6 arrangements of 1, 3, 5
6 arrangements of 2, 3, 4
3 arrangements of 1, 4, 4 - {(1, 4, 4), (4, 1, 4), (4, 4, 1)}
3 arrangements of 2, 2, 5
1 arrangement of 3, 3, 3
25 possible arrangements

Probability of scoring 9 with 3 dice
$= \frac{25}{216} = 0.116$ to 3 decimal places.

Total of 10

6 arrangements of 1, 5, 4
6 arrangements of 1, 3, 6
6 arrangements of 2, 3, 5
3 arrangements of 2, 4, 4
3 arrangements of 2, 2, 6
3 arrangements of 3, 3, 4
27 possible arrangements.

Probability of scoring 10 with 3 dice
$= \frac{27}{216} = 0.125$
so a score of 10 is more likely than a score of 9.

It is not necessary for younger learners to introduce the word 'arrangements' – you could simply call this the number of ways of getting the score.

DIAGNOSTIC ASSESSMENT This should take about 5–10 minutes.

Write the question on the board, say to the class:

"Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".

1. Notice how the learners respond. Ask a learner who gave answer A to explain why he or she gave that answer. DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
2. It is important for learners to explain the reasons for their answers.
   Putting thoughts into words may help them to gain better understanding and improve their communication skills.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. Ask the class to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

How many ways can you get a score of 8 when you throw two dice?

A. 3 B. 4 C. 5 D. 6

The correct answer is: C The scores are (6, 2) (5, 3) (4, 4) (3, 5) (2, 6)

Possible misconceptions:

A. Students might have thought of 3 combinations 8 = 6 + 2 = 5 + 3 = 4 +4 but not that there are two ways of scoring 5 and 3 and two ways of scoring 6 and 2.

B. or D. Maybe just guesses.

Why do this activity?

This activity gives learners an opportunity for comparing experimental and theoretical probabilities. It also involves thinking systematically about how many different combinations of numbers sum to 9 or sum to 10. Often permutations and combinations are taught formally to older learners who would find the ideas easier if they had met them in simple problem solving situations at an earlier age (without using the technical terms).

Learning objectives

In doing this activity students will have an opportunity to:

- deepen understanding of probability of 2 and 3 successive events;
- to build mathematical thinking and problem solving skills.

Generic competences

In doing this activity students will have an opportunity to build problem solving skills, including the ability to identify and consider all possible cases, and to work systematically.

Suggestions for teaching

Two dice. Ask learners "If I throw two dice am I more likely to get a sum of 9 or 10?" If you have dice available you can throw them 10 times and record the scores. Ask for a show of hands for those who think 9 more likely and those who think 10 more likely.

For example your results might be: (3,6); (1,1); (6,5); (5,5); (2,1); (3,3); (4,3); (4, 6); (5,3); (2,5) so the experiment gives 9 with a relative frequency of 1 in 10 and it gives 10 with a relative frequency of 2 in 10. As a result of this experiment learners may think the probability of getting a 10 is higher than of getting a 9. To believe that experiments give a true indication of actual probabilities is a common misconception and we have to be careful not to jump to the wrong conclusions. It is important for learners to know this and that it also applies to real life applications.

If you have sufficient dice each pair or small group of learners could throw the dice 10 times and then the class could collect and compare ALL the results for a larger number of experiments.

So far these results only give **an approximate experimental probability** based on relative frequency. In most real-life applications like clinical trials in medical research and opinion polls before parliamentary elections, this is the only possibility.

In dice throwing games it's possible to calculate an **exact theoretical probability**.

Ask the learners how many outcomes are there altogether when you throw two dice (how many in the sample space). Then ask them to list the outcomes that give the outcome 9 and then for 10. Give them some thinking time to work out the number of outcomes and the probabilities, then ask them to share their findings in a whole class discussion.

For 3 dice you might ask the class to give you 3 numbers (from 1 to 6) that add up to 9 for example 1 + 2 + 6. Ask: "Can anyone suggest 3 different numbers? " and "What about other sets of 3 numbers?" ... Continue until you have all 6 number triples that add up to 9. Then do the same for 10.

Ask the class to image three dice, red, blue and green, and to tell you the different ways that they could score 9 from the numbers 1, 2 and 6 on the dice. Write them in the table.

Now ask the learners to work out the different ways you could get a 9 by throwing a die 3 times (or using dice of 3 different colours) with other number triples. Get different pairs or groups of learners each to write down the numbers, in different orders, that add up to 9 and to 10 : one group to do 1,3,5 another 2,3,4 etc … for 9 and another group to do 1,3,6 etc. for 10. Ask learners to come to the board to write the number triples in the table as you have done for 1, 2 and 6.

SCORES OF 9		**SCORES OF 10**	
1,2,6	(1,2,6); (1,6,2); (2,1,6); (2,6,1); (6,1,2); (6,2,1)	1,3,6	
1,3,5		1,4,5	
2,3,4		2,2,6	
1,4,4		2,3,5	
2,2,5		2,4,4	
3,3,3		3,3,4	
TOTAL NUMBER OF DIFFERENT WAYS OF GETTING THE SCORE (ARRANGEMENTS)			

When they have recorded all their answers on the board, count the total number of ways of getting each score.

Finally ask the class how many outcomes altogether (6×6×6=216) and ask them to complete the following:

Probability of 9 = $\frac{?}{216}$ = ? as a decimal, and

Probability of 10 = $\frac{?}{216}$ = ? as a decimal.

Key questions

- Can you write those numbers in a different order?
- How many ways could the same numbers come up in different orders.
- Can you write down the different ways of getting 9 with those two (three) numbers?
- Can you write down the different ways of getting 10 with those two (three) numbers?
- How many outcomes altogether when you throw 2 dice? Why is it 36?
- How many outcomes altogether when you throw 3 dice? Why is it 216?

Follow up

At Least One https://aiminghigh.aimssec.ac.za/years-8-10-at-least-one/

Two Aces https://aiminghigh.aimssec.ac.za/years-9-12-two-aces/

Spin High Spin Low https://aiminghigh.aimssec.ac.za/years-3-8-spin-high-or-low/

In a Box https://aiminghigh.aimssec.ac.za/years-6-12-in-a-box/

In the Bag https://aiminghigh.aimssec.ac.za/years-6-12-in-a-box/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and school years up to Secondary 5 in East Africa.
New material will be added for Secondary 6.
For resources for teaching A level mathematics (Years 12 and 13) see https://nrich.maths.org/12339
Mathematics taught in Year 13 (UK) & Secondary 6 (East Africa) is beyond the SA CAPS curriculum for Grade 12

	Lower Primary Approx. Age 5 to 8	Upper Primary Age 8 to 11	Lower Secondary Age 11 to 15	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13