

ONE TO EIGHT PUZZLE

Which of the following products are close to 1368:

20×40 20×50 20×70 40×50 40×70 50×70 ?

$$** \times ** = 1368$$

$$5* \times 6* = ****$$

$$52 \times ** = ****$$

In these three multiplication sums, using the digits 1 to 8 once and only once each time, can you replace the * symbols and make the multiplications correct?

Can you explain how you found the answers?

HELP

You can use a calculator. If you work in pairs then one person can use the calculator while the other writes down the results. You can take it in turns to do the two tasks.

Work out 20×40 20×50 20×70 40×50 40×70 50×70 . Then start working on the problem $** \times ** = 1368$ and discuss with each other ways that you might solve it .

NEXT

Try the problem Funny Factorisation <http://nrich.maths.org/740>

Notes for teacher

Why do this activity?

In seeking to be sure that they have found all possible solutions and better ways to record their work learners will see the need to develop strategies that help them to narrow down their options so that they can work systematically to find solutions.

This problem helps learners to develop a better understanding of place value and to develop and apply an understanding of how prime numbers form the building blocks of all whole numbers.

It is good to have three similar problems available to challenge those who work faster but each problem here offers a new and slightly different challenge.

Suggestions for teaching

Let the learners use calculators. Working in pairs one learner could use the calculator while the other writes down the results and they can take it in turns to do the two tasks. It is important for the teacher not to tell the learners how to solve the problems but to let learners start working on the first problem $** \times ** = 1368$ discussing with each other ways they might solve it themselves.

In any problem-solving activity the first step is to understand the problem so make sure that the learners know that, because the digits are used once and only once, they must use the digits 2, 4, 5 and 7 to replace the * symbols and then let them experiment and try out different possibilities.

You could give the hint that it might help if they decide which of the following products are close to 1368: 20×40 20×50 20×70 40×50 40×70 50×70 .

Alternatively you could give them the hint that it might help if they find the prime factors of 1368.

If some learners develop good strategies for solving the first problem then they could be asked to present their ideas to the class.

Then suggest that the learners try the second problem $5^* \times 6^* = ****$ and make sure they know that they should use the digits 1 to 8 without repeating any digit. They will almost certainly try different possibilities in some random order. Let them work on the problem for a while and then ask them if they can rule out any of the **units** digits in 5^* and 6^* . Later ask if they can think up a good system for checking that they have found all the solutions.

Some learners in the class may only succeed with the first of the three problems but they should feel encouraged if they understand how to do it without the teacher telling them what to do.

Key questions

1. Can you find the prime factors of 1368? Does that help with this problem?
2. Which digits are not in 1368? (Answer 2, 4, 5 and 7)
3. OK using 2, 4, 5 and 7, can you find what the tens digits could be in $** \times **$? (answer 2 & 4, 2 & 5, 2 & 7, 4 & 5, 4 & 7 and 5 & 7.)
4. Which of the products (20×40 20×50 20×70 40×50 40×70 50×70) is close to 1368?
5. If you work out $5^* \times 6^*$ for different values of the *s what would you expect the thousands digit to be in the answer?
6. If you work out $5^* \times 6^*$ what units digits will give a repeated digit in the answer?
7. Can you make a list of all the possible products to try out? (There should be 10 pairs in the list).
8. If you work out $52 \times **$ what should the tens digit be in $**$?
9. If you work out $52 \times **$ what numbers can you try for the $**$ that won't repeat the 5 or the 2 in the answer?
10. Can you make a list of all the possible products to try out? (There should be 10 pairs in the list)

Follow up

Try the problem Funny Factorisation <http://nrich.maths.org/740>

Solutions

$$** \times ** = 1368$$

Consider the products 20×40 20×50 20×70 40×50 40×70 50×70 .

The closest to 1368 are: $20 \times 50 = 1000$ and $20 \times 70 = 1400$. The other products are too small or too large. This shows that the solution must be $2^* \times 5^* = 1368$.

The only other digits are 4 and 7 and the only possible cases to be checked are

24×57 and 27×54 . The solution is $24 \times 57 = 1368$.

Another method is to find the prime factors:

$$1368 = 2 \times 684 = 2 \times 2 \times 342 = 2 \times 2 \times 2 \times 171 = 2 \times 2 \times 2 \times 3 \times 57 = 2^3 \times 3 \times 57.$$

This shows that 57 must be one of the 2-digit factors and so the other one is $2^2 \times 3 = 24$.

$$5^* \times 6^* = ****$$

The product is between $51 \times 62 = 3162$ and $58 \times 69 = 4012$ but these are not solutions. Use the digits 1, 2, 3, 4, 7 and 8.

The last digits cannot be 1 or the pairs 2 and 3 or 2 and 8 or 7 and 8 so there are exactly 10 cases to test: 52×64 , 54×62 , 52×67 , 57×62 , 53×64 , 54×63 , 54×67 , 57×64 , 54×68 and 58×64 .

The solution is $58 \times 64 = 3712$.

$$52 \times ** = ****$$

Use the digits 1, 3, 4, 6, 7 & 8.

The last digit cannot be 1 or 6 because that would give 2 as the units digit in the product. Also $52 \times 19 < 1000$ so 1 cannot be the tens digit.

Try the cases: 52 x all the following two digit numbers:

34, 36, 37, 38, 36, 47, 48, 67, 68 and 78.

The solution is $52 \times 34 = 1768$.