African Institute for Mathematical Sciences
Schools Enrichment Centre
AIMSSEC

Grades 4 to 7 Same Sweets

A bag has a large number of green, white, orange, yellow and red sweets with equal numbers of each colour (5 flavours).
You pick sweets from the bag without looking.
If you pick 2 sweets what different combinations of colours can you get?
If you pick 2 sweets how likely are you to pick two of the same colour?
If you pick 6 sweets what is the probability that 2 are the same colour?

There are 25 different possible combinations of colours with 5 pairs the same colour:
(Key: G denotes green, W for white, O for orange, Y for yellow and R for red).

GG	GW	GO	GY	GR
WG	WW	WO	WY	WR
OG	OW	OO	OY	OR
YG	YW	YO	YY	YR
RG	RW	RO	RY	RR

If you pick 2 sweets the probability that two sweets are the same colour is $\frac{5}{25} = \frac{1}{5} = 0.2$

If you pick 6 sweets sweets then it is certain that 2 are the same colour (by the pigon hole principle). The probability is 1.

In the last two columns one probability is 1 minus the other probability because one is the probability that an event happens and the other is the probability of that event not happening.	Number of sweets you pick out without looking	Probability that no two sweets picked are the same colour.	Probability that the two sweets picked are the same colour
	2	4/5 = 0.80	0.20
	3	4/5 x 3/5 = 0.48	0.52
	4	4/5 x 3/5 x 2/5 = 0.192	0.818
	5	4/5 x 3/5 x 2/5 x 1/ 5 = 0.00384	0.9616
	6	0	1

Notes for Teaching

Why do this problem

It provides a context to which learners can easily relate where they can discover answers for themselves and not need to use a formula. Learners have to work systematically to find out that there are 25 different colour combinations. This could be where you stop with Grade 4 or 5.

Each colour combination for 2 sweets is equally likely. With less information than given on this sheet you would need to discuss with the class whether it is reasonable to assume this.

With a little guidance from the teacher learners can then experience the thinking about probability that attaches a value of 1/25 = 0.04 or 4% to the probability for each colour combination. Given enough time to explore and discuss the idea, most learners at Grade 6 or 7 will be able to make the next step to realising that the probability of picking 2 sweets the same colour is 5/25 = 0.2 = 20%.

The reasoning in this problem is exactly like the reasoning for solving the famous Birthday Problem and high fliers can be introduced to the Birth-month problem and possibly to the Birthday Problem.

Suggestions for teaching:

It is important to let learners find out for themselves how many colour combinations there are and find their own way to represent and explain it. Ideally they will discover a neat way to organise the representation in something like the 5 by 5 layout above but the teacher should let the learners discover this for themselves. At first it is likely that they will just record all the different colour combinations they can think of in no special order. Give the class plenty of time to discuss how to record the results to be sure they have found all possible combinations. This works well for a 'One – two – four – more' lesson where learners work individually, then in pairs, then in fours, then the whole class discusses the problem with learners presenting their ideas on the chalkboard.

Key question 7 requires an application of the pigeon hole principle but again the teacher should not give the rules at the start but only at the end of the lesson when the learners have discovered the logic behind the concepts for themselves.

Key Questions

1. Can you find a neat way to record all the colour combinations?
2. Are you sure that you have found all the possibilities? How do you know?
3. Are all the different combinations equally likely? How do you know?
4. What is the probability of each colour combination?
5. How many combinations are there with 2 sweets the same colour?
6. How likely is it that you pick 2 sweets the same colour?
7. If you picked 6 sweets what is the probability that there would be 2 the same colour?

Possible Support

One of the 'golden rules' of problem solving is to work on simple cases when a problem seems difficult. For learners who find this problem difficult simplify the problem for them to a bag of sweets with only 2 colours (4 colour combinations as with the sexes of children in families of two children) . After that they can progress to a bag of sweets with 3 colours (9 colour combinations) and then a bag of sweets with 4 colours (16 colour combinations).

Possible Extension

Try the 'Same Birth-month' and the 'Same Birthday' problems on http://aiminghigh.aimssec.org/

In summary, at the end of the lesson

RULES OF PROBABILITY

1. All events have a probability between 0 and 1.
2. The probability of an event NOT happpening is 1 – the probability the event happens.
3. The probability of an event is the fraction: $\frac{\text{the number of ways the event can happen}}{\text{the total number of different possible events}}$
4. **PIGEON HOLE PRINCIPLE, explained by an example**: If there are 30 post boxes, and more than 30 letters are put in the boxes, then at least one box must have 2 or more letters put in it.