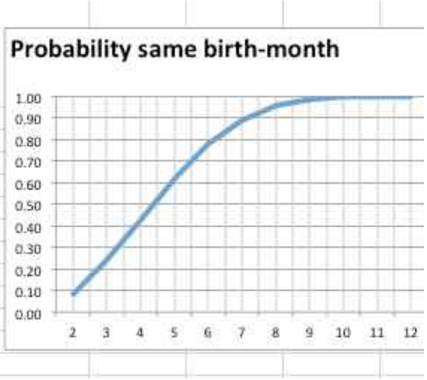
African Institute for Mathematical Sciences
Schools Enrichment Centre
AIMSSEC

Grades 8 to 10 Same Birth-month

Number of people in group	Probability 2 people have same birth-month
2	0.08
3	0.24
4	0.43
5	0.62
6	0.78
7	0.89
8	0.95
9	0.98
10	0.996
11	0.999
12	0.999
13 or more	1



The probability that two people in a group have birthdays in the same month depends on the size of the group.

What if the group has more than twelve people?

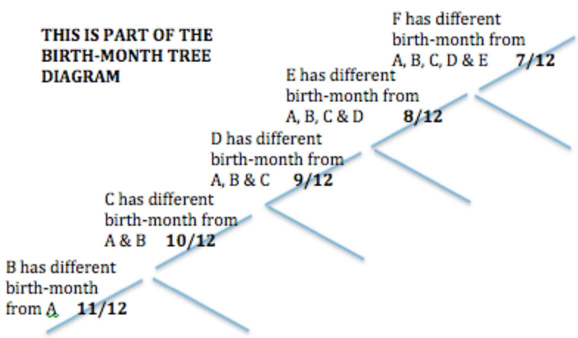
What about a group of 2 people? What is the probability they have birthdays in different months?

What about a group of 3 people? Let's call them A, B and C. Suppose A and B have birthdays in different months. What is the probability that C's birthday is in a different month from A and B?

What is the probability that A, B and C all have birthdays in different months?

What is the probability of two of them having birthdays in the same month?

Solution: If there are more than 12 people then, by the pigeon-hole principle, it is certain that there will be at least one pair with birthdays in the same month. The probability is 1.



Suppose there are 2 people, A and B. Whatever month A is born in then the probability that B was born in a different month is 11/12.

With a group of 3 people A, B and C: suppose A and B have birthdays in different months then there are 10 other months and the probability that C's birthday is in a different month from A and B is 10/12.

To find the probability that A, B and C have different birth months we multiply the probability that A & B have different birth months by the probability that C is born in a different month from A and B.

Pr(A, B and C are born in different months)
= Pr (A & B are born in different months) x Pr(C is born in a 3rd month given that A & B are born in different months)

$$= \frac{11}{12} \times \frac{10}{12} = \frac{110}{144} = 0.76$$

Number of people in group	Probability they all have birthdays in different months (given to 2 decimal places)	Probability two have birthdays in the same month
2	11/12=0.92	1 - 0.92 = 0.08
3	(11/12)x(10/12) = 0.76	1 – 0.76 = 0.24
4	(11/12)x(10/12)x(9/12) =0.57	1 – 0.57 = 0.43
5	(11/12)x(10/12)x(9/12)x(8/12)=0.38	1 – 0.38 = 0.62
6	(11/12)x(10/12)x(9/12)x(8/12)x(7/12)=0.22	1 – 0.22 = 0.78
7	(11/12)x(10/12)x(9/12)x(8/12)x(7/12)x(6/12)=0.11	1 – 0.11 = 0.89

To get the probability that 4 people are all born in different months we use the same reasoning and multiply 110/144 by 9/12 (as illustrated in the section of the tree diagram shown above). The calculations for up to 7 people are shown in the table.

These calculations, done on a spread-sheet, are shown above with a graph of the probabilities for groups from 2 people to 13 people.

Notes for Teaching

Why do this problem

The Birth-month problem provides a context to which learners can easily relate where they can engage in problem solving and discover answers for themselves.

One of the important problem solving skills is being able to generalise and extend a method that works in a simple case. As given this question only asks about a group of 3 people. The teacher can decide whether to ask the learners to try to solve it for a group of 4 people, and then perhaps to continue extending the argument for groups of 5, 6, 7, 8, 9, 10, 11 and 12 people.

High fliers can be introduced to the Birthday Problem where the reasoning is the same.

Suggestions for teaching:

The teacher could start by asking if this is the sort of problem where the exact answer can be found by doing calculations or is it the sort of problem (like popularity surveys and medical trials) where it is only possible to find approximate answers by doing surveys and experiments. As this problem is of the first type there is no point in wasting time on surveys.

It is important to let learners discuss the problem and suggest ways of solving it themselves. You could use the 'One – two – four – more' strategy where learners work individually, then in pairs, then in fours, then the whole class discusses the problem with learners presenting their ideas on the chalkboard. That way the teacher can help all the groups learners to have some success with the problem while offering more of a challenge to those who succeed in finding the answer quite quickly. While everyone starts with the same problem, the high fliers can be offered the additional challenge of extending the argument to groups of 4 or more people.

Drawing a whole tree diagram is a lot of work and not really necessary. If learners decide to use a tree diagram the teacher might like to help them to focus on the bit they need to use and point out that they don't need to draw the whole diagram.

Key Questions

1. How many different months are there?
2. Can 13 people all be born in different months?
3. For 2 people what is the probability that the second person is born in the SAME month as the first person? Why?
4. For 2 people what is the probability that the second person is born in a DIFFERENT month from the first person? Why?
5. Suppose two people are born in different months, what is the probability that a third person is born in a DIFFERENT month from both of the first two people? Why?
6. If you know the probabilities of two events how do you work out the probability that both events happen?
7. If you have worked out the probability that an event happens how do you find the probability that the event does NOT happen?
8. If you have worked out the probability that 3 people are all born in different months, how do you find the probability that this does not happen?

Possible Support

One of the 'golden rules' of problem solving is to work on simple cases when a problem seems difficult. Learners who find this problem difficult could be given the 'Same Sweets' problem on http://aiminghigh.aimssec.org/ which is is a simple version of the Birth-month problem with 5 different possibilities rather than 12.

Possible Extension

Try the 'Same Birthday' problem on http://aiminghigh.aimssec.org/ which is an extension of this problem with 366 different possibilities (days in a year) rather than 12 different possibilities, the months in a year.

In summary, at the end of the lesson

RULES OF PROBABILITY

1. All events have a probability between 0 and 1.
2. The probability of an event NOT happpening is 1 – the probability the event happens.
3. The probability of an event is the fraction: $\dfrac{\text{the number of ways the event can happen}}{\text{the total number of different possible events}}$
4. If S and T are two events,

Probability(S and T) = Probability(S) × Probability (T given that S has already happened).

5. **PIGEON HOLE PRINCIPLE, explained by an example**: If there are 30 post boxes, and more than 30 letters are put in the boxes, then at least one box must have 2 or more letters put in it.