African Institute for Mathematical Sciences
Schools Enrichment Centre
AIMSSEC

Grades 7 to 10 One to Eight (757)

Which of the following products are close to 1368:

20 x 40 20 x 50 20 x 70 40 x 50 40 x 70 50 x 70?

**** × ** = 1368**

5* × 6* = ****

52 × ** = ****

In these three multiplication sums, using the digits 1 to 8 once and only once each time, can you replace the * symbols and make the multiplications correct?

Can you explain how you found the answers?

Solutions:

**** x ** = 1368**

Consider the products 20x40, 20x50, 20x70, 40x50, 40x70 and 50x70.

The closest to 1368 are: 20 x 50 = 1000 and 20 x 70 = 1400. The other products are too small or too large.

This shows that the solution must be 2*x 5* = 1368.

The only other digits are 4 and 7 and the only possible cases to be checked are 24x57 and 27x54.

The solution is 24 x 57 = 1368.

Another method is to find the prime factors:

1368 = 2 x 684 = 2 x 2 x 342 = 2 x 2 x 2 x 171 = 2 x 2 x 2 x 3 x 57 = 2^3 x 3 x 57.

This shows that 57 must be one of the 2-digit factors and so the other one is 2^2x3=24.

5* x 6* = ****

The product is between 51 x 62 = 3162 and 58 x 69 = 4012 but these are not solutions.

Use the digits 1, 2, 3, 4, 7 & 8.

The last digits cannot be 1 or the pairs 2 & 3 or 2 & 8 or 7 & 8 so there are exactly 10 cases to test:

52x64, 54x62, 52x67, 57x62, 53x64, 54x63, 54x67, 57x64, 54x68, 58x64.

The solution is 58x64=3712.

52 x ** = ****

Use the digits 1, 3, 4, 6, 7 & 8.

The last digit cannot be 1 or 6 because that would give 2 as the units digit in the product.

Also 52x19<1000 so 1 cannot be the tens digit.

Try the cases: 52 x all the following two digit numbers:

34, 36, 37, 38, 36, 47, 48, 67, 68 and 78.

The solution is 52x34=1768.

Notes for Teaching

Why do this problem

This problem helps learners to develop a better understanding of place value and to develop and apply an understanding of how prime numbers form the building blocks of all whole numbers. In seeking better ways to record their work and to be sure that they have found all possible solutions learners will see the need to develop strategies that help them to narrow down their options so that they can work systematically to find solutions.

It is good to have three similar problems available to challenge those who work faster but each problem here offers a new and slightly different challenge .

Suggestions for teaching:

Let the learners use calculators. Working in pairs one learner could use the calculator while the other writes down the results and they can take it in turns to do the two tasks. It is important for the teacher not to tell the learners how to solve the problems but to let learners start working on the first problem ** x ** = 1368 and discussing with each other ways they might solve it themselves.

In any problem solving activity the first step is to understand the problem so make sure that the learners know that the digits they must use to replace the * symbols are 2, 4, 5 and 7 and them let them experiment and try out different possibilities. You could give them the hint that it might help if they find the prime factors of 1368.

Alternatively you could give the hint that it might help if they decide which of the following products are close to 1368:

20 x 40 20 x 50 20 x 70 40 x 50 40 x 70 50 x 70.

If some learners develop good strategies for solving the first problem then they could be asked to present their ideas to the class.

Then suggest that the learners try the second problem 5* x 6* = **** and make sure they know that they should use the digits 1 to 8 without repeating any digit. They will almost certainly try different possibilities in some random order. Let them work on the problem for a while and then ask them if they can rule out any of the **units** digits in 5* and 6*. Later ask if they can think up a good system for checking that they have found all the solutions.

Some learners in the class may only succeed with the first of the three problems but they should feel encouraged if they understand how to do it without the teacher telling them what to do.

Key Questions

1. Can you find the prime factors of 1368? Does that help with this problem?
2. Which digits are not in 1368? (Answer 2, 4 ,5 and 7)
3. OK using 2, 4, 5 and 7, can you find what the tens digits could be in ** x **? (answer 2 & 4, 2 & 5, 2 & 7, 4 & 5, 4 & 7 and 5 & 7.)
4. Which of the products (20 x 40, 20 x 50, 20 x 70, 40 x 50, 40 x 70, 50 x 70) is close to 1368.
5. If you work out 5* x 6* for different values of the *s what would you expect the thousands digit to be in the answer?
6. If you work out 5* x 6* what units digits will give a repeated digit in the answer?
7. Can you make a list of all the possible products to try out? (There should be 10 pairs in the list).
8. If you work out 52x** what should the tens digit be in **?
9. If you work out 52x** what numbers can you try for the ** that won't repeat the 5 or the 2 in the answer?
10. Can you make a list of all the possible products to try out? (There should be 10 pairs in the list

Possible Support

Don't offer the hints to everyone but only to the learners who really need support. The teachers role is to try to help the learners to be more independent and not to 'spoon feed' them. Again the key questions listed here give hints and as your learners do more problem solving and develop skills and confidence you will need to use questions like these less and less for guidance.

Possible Extension

Try the problem Funny Factorisation http://nrich.maths.org/740