AIMSSEC

AFRICAN INSTITUTE FOR MATHEMATICAL SCIENCES
SCHOOLS ENRICHMENT CENTRE (AIMSSEC)
AIMING HIGH

WHY 24?

1. Write down three consecutive whole numbers none of which is a multiple of three. If you can't do it, explain why.

2. Multiply any two consecutive even numbers together. Why is the product always a multiple of 8?

3. Take any prime number greater than 3, square it, subtract one and divide by 24.

Make a statement about what you notice about squaring prime numbers, subtracting one and dividing by 24 (a conjecture) and prove that what you say is always true.

Help

You have to make a statement about what you notice about squaring prime numbers, subtracting one and dividing by 24 (a conjecture) and prove that **what you say is true for all prime numbers greater than 3**.

FIRST CARD IN PROOF Let p be a prime number greater than 3.	*The first and last cards are in the right places. Rearrange the other cards so that each statement follows from the one before it.*
p is an odd number, so p - 1 and p + 1 must both be multiples of 2.	(p - 1)(p + 1) is the product of a multiple of 2 and a multiple of 4, so must be a multiple of 8.
(p - 1) and (p + 1) are consecutive even numbers so either (p - 1) or (p + 1) must be a multiple of 4.	(p - 1), p, and (p + 1) are consecutive numbers.
p is prime and greater than 3 so cannot be a multiple of 3.	(p - 1)(p + 1) is a multiple of both 8 and 3, so (p - 1)(p + 1) is a multiple of 24.
The expression $p^2 - 1$ can be factorised as (p - 1)(p + 1)	Either (p - 1) or (p + 1) must be a multiple of 3, so the product (p - 1)(p + 1) must be a multiple of 3.
If I have three consecutive numbers, one of them must be a multiple of 3.	**LAST CARD IN PROOF** Therefore for any prime number p greater than 3, $p^2 - 1$ is a multiple of 24.

Try it for the prime number 7 and you will find $7^2 - 1 = 49 - 1 = 48$ which is a multiple of 24.
What about $11^2 - 1$? What about $13^2 - 1$?

This is a Proof Sorter Activity. If possible work as a group and do not move to the next step in the proof until everyone understands everything up to that step.

Cut out the larger cards on the next page and share them between the members of the group. Each person should study their card(s) and then explain to everyone else what it means. Use different values of p like p = 7 or 11 or 13 or 17 to make sense of the statements and convince everyone in the group that the statement on the card is true for all prime numbers p greater than 3.

Arrange the cards in order to give a proof of the conjecture.

Next

Write down your own explanation of the proof of the conjecture.

How many ways can you combine 4 whole numbers from 1, 2, 3 … 10 by addition, subtraction, multiplication and division to get the answer 24? Numbers can be repeated. For the numbers 4, 7, 8, 8, a solution is $(7 - (8 \div 8)) \times 4 = 24$ and for 3, 6, 9 and 10 here are 2 solutions: $6 \times (3+10-9) = 24$; $(6 \times 9) - (3 \times 10) = 24$.

Make your own 24 Game. Make lots of cards, each with 4 numbers from 1 to 10 that can be combined by +, -, × and ÷ to get the answer 24. To play the game use a timer, give one card to the first player. S/he keeps the card if she can make 24, and if not the first of the other players to make 24 gets the card, or it is shuffled back into the pack if nobody can make 24. When all the cards have been used the player with most cards wins the game. Alternatively, use an ordinary pack of playing cards, make aces take the value 1 and honour cards the value 10. Deal 4 cards to the first player and proceed as before.

FIRST CARD IN PROOF Let p be a prime number greater than 3.	*The first and last cards are in the right places. Rearrange the other cards so that each statement follows from the one before it.*
p is an odd number, so p - 1 and p + 1 must both be multiples of 2.	(p - 1)(p + 1) is the product of a multiple of 2 and a multiple of 4, so must be a multiple of 8.
(p - 1) and (p + 1) are consecutive even numbers so either (p - 1) or (p + 1) must be a multiple of 4.	(p - 1), p, and (p + 1) are consecutive numbers.
p is prime and greater than 3 so cannot be a multiple of 3.	(p - 1)(p + 1) is a multiple of both 8 and 3, so (p - 1)(p + 1) is a multiple of 24.
The expression $p^2 - 1$ can be factorised as (p - 1)(p + 1)	Either (p - 1) or (p + 1) must be a multiple of 3, so the product (p - 1)(p + 1) must be a multiple of 3.
If I have three consecutive numbers, one of them must be a multiple of 3.	**LAST CARD IN PROOF** Therefore for any prime number p greater than 3, $p^2 - 1$ is a multiple of 24.