Title: WRIT LARGE (Grade 9)

WRIT LARGE

Suppose you had the never ending task of writing out all the natural numbers:

1, 2, 3, 4, 5.... and so on.

What would be the 1000th digit you would write down?

In what number would it occur?

What number would contain the millionth digit?

When would you have written six for the 6000th time?

Solution

The 1000th digit is the 3 of 370

From 1 to 99 there are 189 digits.

From 100 to 200 there are 303 digits.

From 200 to 300 there are 300 digits.

From 1 to 300 there are 792 digits.

So the 1000th digit lies between 300 and 400 and it is the 208th digit counting from the 3 in 301.

$208/3 = 69.333$

So the 1000th digit is the 3 in 370.

The millionth digit is the initial 1 in 185185.

1 digit	1 to 9	9 x 1	9	
2	10 to 99	90 x 2	180	
3	100 to 999	900 x 3	2700	
4	1000 to 9999	9 000 x 4	36 000	
5	10 000 to 99999	90 000 x 5	450 000	
		total	**488 889**	

$1\,000\,000 - 488\,889 = 511\,111$

So we need some 6 digit numbers to give a total of 511 111 digits.

$511\,111/6 = 85\,185.1666$

$488\,889 + 6(85\,185) = 488\,889 + 511\,110 = 999\,999$

Starting from 100 000 the 85185th number is 185184

so the millionth digit is the first digit of 185185.

The six thousandth six

From 1 to 100 you write '6':	10 times as the units digit 10 times at the tens digit	20 times
From 1 to 1000 you write '6':	100 times as the units digit 100 times as the tens digit 100 times as the hundreds digit	300 times
From 1 to 10000 you write '6'	1000 times as the units digit 1000 times as the tens digit 1000 times as the hundreds digit 1000 times as the thousands digit	4000 times
From 10000 to 15999 you write '6':		$6 \times 300 = 1800$ times
From 16000 to 16099 you write '6':		120 times
From 16100 to 16159 you write '6':		66 times
From 16160 to 16165 you write '6':		12 times
From 1 to 16165 you write the digit '6' altogether		5998 times

So you write '6' for the 6000th time as the second '6' in the number 16166.

Notes for teachers

Why do this activity?

This activity gives a chance for learners to develop mathematical reasoning where the focus is on thinking logically rather than on trying to remember mathematical facts and techniques. Teachers can use this activity to provide for learners of different abilities.

Learning objective: To develop number sense including a deeper understanding of place value. To develop problem solving skills.

Possible approach

Use the first part of the task for the whole class. Praise all the learners who succeed and then give them the second part as an extension. Give the third part to learners who succeed on the first two parts.

Key questions

How many digits between 1 and 99? Between 100 and 999?...
How many 3 digit numbers are there? 4 digit numbers?...
How many sixes are there in the units? How many in the tens? Hundreds?...
How did you work that out?

Possible extension

Learners who do all three parts successfully can set their own challenges and perhaps challenge their partner.

Possible support

Learners who struggle might first find the 100th digit and then 500th.