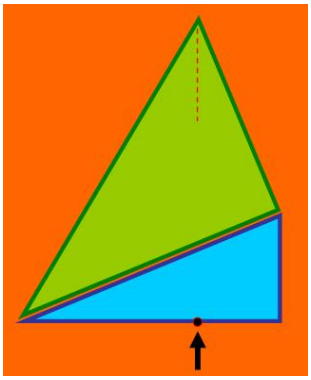
WEDGE ON WEDGE



Two right-angled triangles are connected together as shown in the diagram.

The green triangle has side lengths of 65, 52 and 39 cm and the blue triangle has side lengths of 52, 48 and 20 cm.

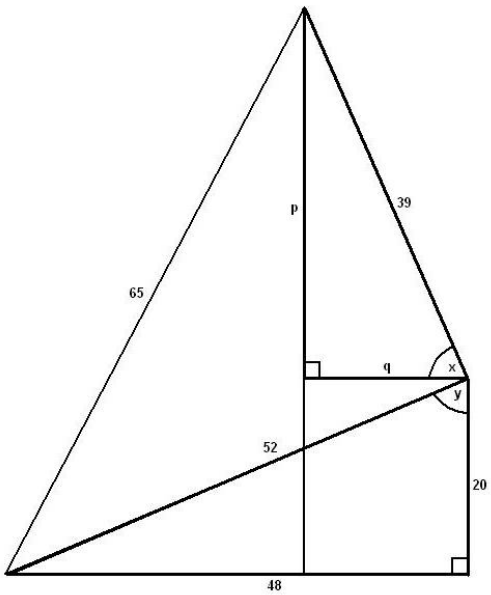
An object is dropped from the top of the green triangle hitting the base by the arrow.

Where does it hit the base of the blue triangle?

In what proportion is the blue base split and how far has the object fallen?

This problem does not require a calculator or any special formula.

HELP



This question is all about similar triangles.

If you need help in drawing the diagram, here it is.

Slowly read the question again and try to understand all the labelling on the diagram.

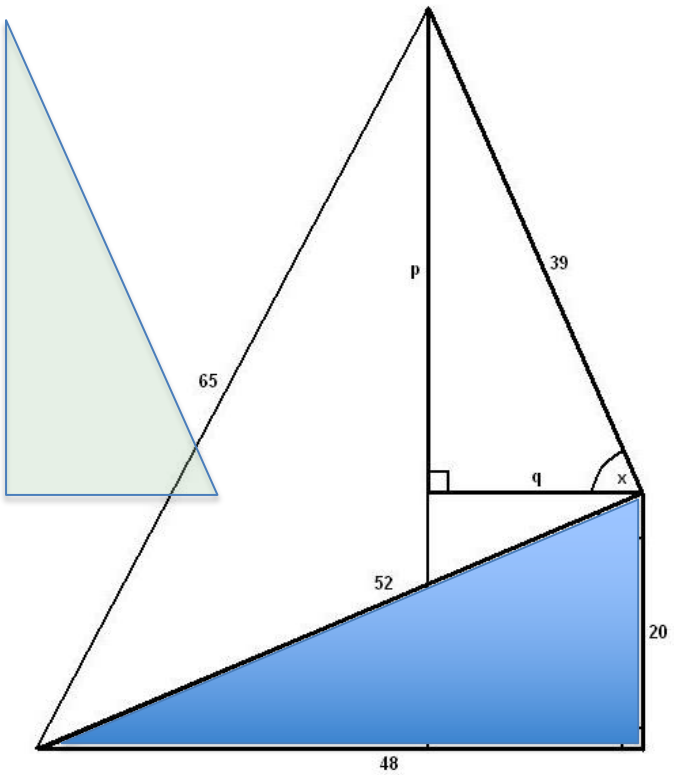
Prove that angles x and y are equal and find similar triangles in the diagram.

NEXT

How would you solve a related problem with numbers that you make up for yourself (or for your partner)? Can you describe a general strategy? Is all data given actually needed?

NOTES FOR TEACHERS

SOLUTION



The shaded triangles are similar because

Angle x = (90 – (90 –y) = y.

So q/39 = 20/52 which gives q = 15.

Also p/39 = 48/52 which gives p = 36.

The blue base of 48 units is split into 33 and 15 units that is in the proportion 11 to 5.

The object has fallen 56 units when it hits the blue base.

Diagnostic Assessment This should take about 5–10 minutes.

1. Write the question on the board, say to the class:
   "Put up 1 finger if you think the answer is A, 2 fingers for B, 3 fingers for C and 4 fingers for D".
2. Notice how the learners responded. Ask a learner who gave answer A to explain why he or she gave that answer and DO NOT say whether it is right or wrong but simply thank the learner for giving the answer.
3. Then do the same for answers B, C and D. Try to make sure that learners listen to these reasons and try to decide if their own answer was right or wrong.
4. **Ask the class again to vote for the right answer by putting up 1, 2, 3 or 4 fingers. Notice if there is a change and who gave right and wrong answers.** It is important for learners to explain the reason for their answer otherwise many learners will just make a guess.
5. If the concept is needed for the lesson to follow, explain the right answer or give a remedial task.

The two triangles below are similar.

What is the value of *y*?

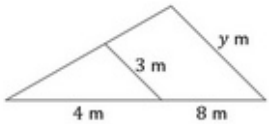


Diagram not drawn accurately.

A 6 m B 7 m C 9 m D 11 m

The correct answer is C: y/3 = 12/4 so y = 9 m.

A. The student has not recognised that y is three times as long, rather than two.

B. and D. It's not obvious why students would choose these answers

https://diagnosticquestions.com

Why do this activity?

This is a geometry rider that uses only similar triangles. This activity is designed to provide learners with an experience of the construction and the mathematical reasoning underpinning the compound angle formula but without any use of trigonometry. It provides an exercise in geometrical reasoning and at the same time paves the way for work on trigonometric formulae.

Learning objectives

In doing this activity students will have an opportunity to:

- revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity of triangles;
- prepare for the introduction of compound trigonometric formulae.

Generic competences

In doing this activity students will have an opportunity to:

- **think mathematically,** reason logically and give explanations and proofs;
- **visualize** and develop the skill of interpreting and creating visual images to represent concepts.

Suggestions for teaching

Ask the learners to read the problem and work on it individually to give them vital practice for writing tests. Only if they cannot get started give individual learners a hint by asking "What sort of triangles do you get if you draw in the line on which the object drops and also put in a horizontal line?"

Only give the diagram in the Help box on page 1 to students who are seriously struggling.

This activity, based on a structure that includes a compound angle, could be a standard post-16 trigonometry question, but it is offered here as an exercise in similar triangles, with side lengths selected to make ratio calculations easy. The objective is to encourage learners to explore the construction on which the compound angle formulae depend.

Eventually students will have a range of formulae to apply to problems like this, but our aim at this stage is to help them spend some time exploring the constructions that make those formulae possible or valid.

Key questions

- What sort of triangles do you get if you draw in the line on which the object drops and put in a horizontal line?
- What do you notice about the angles in the triangles you have created.
- Can you see any similar triangles?

Follow up

More questions on similar triangles:
https://aiminghigh.aimssec.ac.za/years-9-11-kissing-triangles/
https://aiminghigh.aimssec.ac.za/years-11-12-why-the-same/

Note: The Grades or School Years specified on the AIMING HIGH Website correspond to Grades 4 to 12 in South Africa and the USA, to Years 4 to 12 in the UK and up to Secondary 5 in East Africa. New material will be added for Secondary 6.
For resources for teaching A level mathematics see https://nrich.maths.org/12339
Note: The mathematics taught in Year 13 (UK) and Secondary 6 (East Africa) is beyond the school curriculum for Grade 12 SA.

	Lower Primary or Foundation Phase Age 5 to 9	Upper Primary Age 9 to 11	Lower Secondary Age 11 to 14	Upper Secondary Age 15+
South Africa	Grades R and 1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
USA	Kindergarten and G1 to 3	Grades 4 to 6	Grades 7 to 9	Grades 10 to 12
UK	Reception and Years 1 to 3	Years 4 to 6	Years 7 to 9	Years 10 to 13
East Africa	Nursery and Primary 1 to 3	Primary 4 to 6	Secondary 1 to 3	Secondary 4 to 6