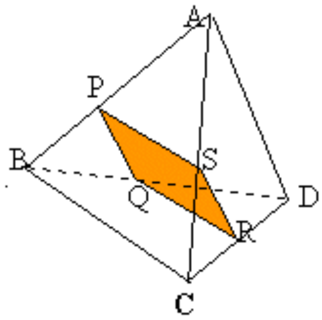

TETRA SQUARE

ABCD is a regular tetrahedron and the points P, Q, R and S are the midpoints of the edges AB, BD, CD and CA.

Prove that PQRS is a square.

What does this tell you about the opposite edges of the tetrahedron?

To help with visualising the problem you might like to make a tetrahedron using six sticks made from paper rolled around pieces of string that you tie together at the vertices.

SOLUTION

As triangle ABC is an enlargement of triangle APS by a scale factor 2, or by similar triangles, we can show that PS is parallel to BC and half the length of BC. Similarly from triangles DQR and DBC we can show that QR is parallel to BC and half the length of BC. **So PS = QR. Similarly PQ = SR**.

We still have to show that PQRS is a square and not a rhombus.

Imagine rotating the tetrahedron about the vertical line through A and the middle of the base BCD.

Then AB ➔ AC so P ➔S and CD ➔ DB so R ➔Q. This shows that PR ➔SQ.

So PR = QS which means that PQRS must be a square.

PS and QR are both parallel to BC and perpendicular to PQ and SR.

PQ and SR are both parallel to AD.

So BC and AD are perpendicular – **the opposite edges of a regular tetrahedron are perpendicular.**

NOTES FOR TEACHERS

Why do this activity?

When the class is working on the Mid-point Theorem for a triangle they could broaden and generalise their mathematical thinking to include enlargements and 3 dimensions. If you don't want to use this activity with the whole class, you might like to give it to the high flyers as an extension activity.

Intended Learning Objectives (Grades 11 and 12)

To develop spatial awareness, visualization skills and mathematical reasoning and communication, through investigating line segments joining the midpoints of two sides of a triangle.

Possible approach

In pairs, the class should make tetrahedra with sticks made from paper rolled around pieces of string that they tie together at the vertices. Then they should work in pairs on the problem and write down their reasons why PQRS is a square and also what they notice about the opposite edges of the tetrahedron. They should try to explain and prove any conjectures they make.

Then they should discuss and compare their findings with another pair of students.

Finally, in a plenary session, several groups of students should be asked to explain their proofs to the class.

Finally write the best proof on the board as a model for the class.

Key questions

Looking at triangle ABC what can you say about PS? Now from triangle DBC what can you say about QR?

What can you prove about the quadrilateral PQRS?

How do you know PQRS is a square and not just a rhombus?

IF PQRS is a square what do you know about the angle between PS and PQ? What does that tell you about the angle between BC and AD? Why is that?

Possible extension

Students should be able to prove (accepting results established in earlier school years):
• that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem);
• that equiangular triangles are similar;
• that triangles with sides in proportion are similar.

This activity extends these concepts to 3 dimensions.

What happens to the quadrilateral PQRS if the points P, Q, R and S are not midpoints but divide the edges AB, BD, CD and CA in the ratio 1 : 2 ?

Possible support

Learners could measure the lengths PQ, QR, RS and SP. You could tie a piece of string around the tetrahedron at the points P, Q, R and S to show that the quadrilateral PQRS is a square.